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ABSTRACT

This study attempted to identify the things that help elementary school students solve verbal problems. Among the types of things included for examination were: the attributes of problems, the tools used, and the problem transformations that were possible. The summary of findings is comprised of 29 diverse types of statements grouped under four headings: (1) Problem Difficulty, (2) Effectiveness of Hint, (3) Calculator Usage, and (4) Special Analyses. These findings included: (1) By sixth grade, most single step problems could be solved by children of all ability levels; (2) For all grade levels and all ability levels, "Personalize" was a helpful problem-solving hint; (3) The pattern of calculator usage by low-ability students was very similar to the calculator usage of average-ability students. Both groups used a calculator more often than high ability students; and (4) Contrary to the results of prior research, problems containing extraneous information which was similar to the problem data were not more difficult than problems with "unlike" data. (MK)

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CHILDREN'S STRATEGIES OF SOLVING VERBAL PROBLEMS

A Final Report to the National Institute of Education

on the Research Supported by

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SE 029 339

CHILDREN'S STRATEGIES OF SOLVING VERBAL PROBLEMS

by Marilyn J. Zweng, University of Iowa

Purpose of the Study

The principal question addressed by this investigation was "What helps elementary school students solve verbal problems?" Several aspects of this question were examined. These include:

1. What attributes of a problem should children focus on for successful solution? The question? The information given? The action? The relationships between the parts and the wholes? The size of the answer?
2. What tools contribute to problem solving success? Drawings? Manipulatives? Calculators?
3. What transformations of a problem help children solve problems? Restatement of the problem with smaller numbers? Restatement of the problem in a less verbal form? Restatement of the problem using a more familiar setting?

Each of these questions addressed the issue of selecting the correct process for problem solution; the role of computational proficiency in problem solving was not investigated. Children in grades three, four, five and six participated in the study.

Interactions among the problem solving "helps", problem types (division problems, subtraction problems, rate problems, and the like), grade level and student problem solving ability were also of interest.

Information on the relative difficulty of various problem types within and across grade levels and within and across ability levels was also gathered, however, this was not the primary concern of the study.

Pilot Study

All problems used in the study were piloted with 24 children, six at each of the grade levels three through six, at Hoover School in Iowa City. During the course of the pilot study which involved a total of 48 interviews, methods of collecting data were also developed and refined.

Design of Study

The Problems

Problem types. Children in grades three through six were asked to solve fifteen typical textbook-type problems in an interview setting. The fifteen problem types were as follows:

Single Step Problems

- Addition
- "Take Away" Subtraction
- Comparative Subtraction (What's the difference?)
- Additive Subtraction (How many more are needed?)
- Multiplication
- Measurement Division - No Remainder
- Measurement Division - With Remainder
- Partition Division - No Remainder
- Partition Division - With Remainder
- Problems with Irrelevant (extraneous) Data

Multiple Step Problems

- Problems Using Multiplication (and Addition or Subtraction)
- Problems Using Division (and Addition or Subtraction)
- Problems with Extraneous Data

Rate Problems

Unit Rate a Whole Number (For example, the unit rate for the rate "3 for 15" is "1 for 5")

Unit Rate a Fractional Number (For example, the unit rate for the rate "2 for 15" is "1 for 7½")

The continuous-discrete factor. Two different forms were written for comparative and additive subtraction problems, the four division problems and the two rate problems. For each of these problem types, one form was about continuous quantities such as string, miles or minutes and the other form was about discrete units, as for example sticks of candy, pencils and the like. When the problems were piloted there appeared to be differences between the strategies children used to solve problems about discrete units and the strategies used to solve problems about continuous quantities, so it was decided to control for this factor. At each grade level approximately half of the children received a given problem type in the continuous setting and half received it in a discrete quantities setting. Furthermore the settings were alternated, that is, if a child received a continuous comparative subtraction problem, the additive subtraction problem he received was in the discrete form.

The extraneous data factors. There were also two forms of the single step irrelevant data problem. Form "u" (for unlike) contained extraneous information which was in a different unit than the other data in the problem. For example, if the problem was about cents, the irrelevant fact might be about ounces. In the other form, form "s", the irrelevant information was in the same units as the other problem information.

The problem sets. A different set of problems was written for each grade level, however, for a few problem types, the problems were the same

at two or more grade levels. Every effort was made to avoid "clue words" in the problem statements. For example, "subtraction words" such as "left" and difference", and "addition words" such as "altogether" were not used. Twenty-four problems were written for each grade level, but only fifteen of these problems were administered to any one child. Table 1 summarizes the make-up of each of the problem sets for each grade level. The problems received by an individual subject are indicated by A and B on the table. That is, an A subject received each of the problems marked A, a total of 15 problems; B subjects were administered the 15 problems marked with a B. The complete text of the problems for all four grade levels is provided in Appendix A.

The computation factor. With a few minor exceptions, the computation needed for the solution of the discrete and continuous problems was the same. The difficulty of the computation for a specific problem type generally increased from third through sixth grade. The major exception was rate problems for which the difficulty of the computation was about the same at all four grade levels. Unfortunately, the computation for the two forms of irrelevant data problems differed because we were unable to devise sensible sounding problems conforming to the "same" and "unlike" criteria which both used the same operation. The "u" form used multiplication while the "s" form required subtraction. The computation needed to solve each problem in each of the grade level problem sets is summarized in Table 2.

Administering the Problems

Subjects were interviewed individually. Two sessions from 30 to 45 minutes each were usually needed to complete the 15 problems.

Table 1

Composition of Problem Sets for

A Subjects and for B Subjects

Problem Type	No Special Factors	Type of Quantities		Extraneous Data	
		Cont.	Discrete	Same	Unlike
Single Step Problems					
Addition	AB				
"Take Away" Subtraction	AB				
Comparative Subtraction		A	B		
Additive Subtraction		B	A		
Multiplication	AB				
Measurement Division No Remainder		B	A		
Measurement Division With Remainder		A	B		
Partition Division No Remainder		B	A		
Partition Division With Remainder		A	B		
Extraneous Data				B	A
Multiple Step Problems					
Using Multiplication	AB				
Using Division	AB				
Extraneous Data	AB				
Rate Problems					
Unit Rate a Whole Number		A	B		
Unit Rate a Fractional Number		B	A		

Table 2
Computation Needed to Solve Each Problem Type
By Grade Level

Problem Type	Third	Fourth	Fifth	Sixth
		SINGLE STEP PROBLEMS		
Addition	25 + 19	423 + 354	$13.49 + $8.99	$13.49 + $8.99
"Take away" Subtraction	45 - 18	153 - 42	250 - 89	1275 - 698
Comparative Subtraction				
Continuous	35 - 17	152 - 137	300 - 258	300 - 258
Discrete	65 - 47	152 - 137	310 - 257	310 - 257
Additive Subtraction				
Continuous	25 - 10	98 - 75	1295 - 823	3100 - 1984
Discrete	49 - 35	98 - 75	$129.59 - $82.25	3100 - 1984
Multiplication	3 x 6	12 x 25	28 x 25	120 x 312
Measurement Division No Remainder				
Continuous	24 ÷ 3	732 ÷ 6	864 ÷ 24	864 ÷ 144
Discrete	24 ÷ 3	732 ÷ 6	864 ÷ 24	864 ÷ 144
Measurement Division with Remainder				
Continuous	29 ÷ 8	150 ÷ 8	350 ÷ 15	350 ÷ 15
Discrete	29 ÷ 8	150 ÷ 8	350 ÷ 15	350 ÷ 15
Partition Division No Remainder				
Continuous	24 ÷ 8	432 ÷ 6	432 ÷ 18	432 ÷ 18
Discrete	24 ÷ 8	1650 ÷ 6	432 ÷ 18	432 ÷ 18
Partition Division with Remainder				
Continuous	10 ÷ 4	75 ÷ 30	72 ÷ 27	270 ÷ 24
Discrete	10 ÷ 4	75 ÷ 30	72 ÷ 27	270 ÷ 24

(Table 2, cont'd.)

Problem Type	Third	Fourth	Fifth	Sixth
SINGLE STEP PROBLEMS, continued				
Extraneous Information Similar to Problem Data Unlike Problem Data	24 - 18 4 x 20	480 - 456 8 x 24	1125 - 975 8 x 24	1125 - 975 8 x 24
MULTIPLE STEP PROBLEMS				
Using Multiplication	(4x6) + (4x9) or 4 x (6+9)	(12x15) + (12x18) or 12 x (15+18)	4300 - (7x570)	4300 - (7x570)
Using Division	(27+36) ÷ 9 or (27÷9) + (36÷9)	(60+72) ÷ 12 or (60÷12) + (72÷12)	($37.20-$34.10) ÷ 62	($37.20-34.10) ÷ 62
Extraneous Information Similar to Problem Data	(4x10) - 24	(80x20) - 800	(80x20) - 800	(75x25) - 1250
*RATE PROBLEMS**				
Unit Rate a Whole Number Continuous	$\frac{2}{20} = \frac{10}{x}$	$\frac{2}{20} = \frac{10}{x}$	$\frac{2}{20} = \frac{9}{x}$	$\frac{2}{20} = \frac{9}{x}$
Discrete	$\frac{2}{10} = \frac{3}{x}$	$\frac{3}{15} = \frac{7}{x}$	$\frac{3}{15} = \frac{7}{x}$	$\frac{3}{15} = \frac{7}{x}$

Table 2, cont'd.)

Problem Type	Third	Fourth	Fifth	Sixth
RATE PROBLEMS, continued*				
Unit Rate a Fractional Number				
Continuous	$\frac{2}{15} = \frac{8}{x}$	$\frac{2}{15} = \frac{8}{x}$	$\frac{2}{15} = \frac{8}{x}$	$\frac{2}{15} = \frac{8}{x}$
Discrete	$\frac{2}{15} = \frac{6}{x}$	$\frac{2}{15} = \frac{6}{x}$	$\frac{3}{85} = \frac{12}{x}$	$\frac{3}{85} = \frac{12}{x}$

*These problems could be solved using several different series of operations. The proportion is given to indicate the problem structure rather than to indicate the method children would be expected to use.

Interviews were recorded on audio tape. At the same time, the interviewer recorded the student's problem solving behaviors and performance on a printed form which will be described later.

Order of problem presentation. The fifteen problems, each on a separate sheet of paper, were randomly ordered so that the effect of time of occurrence (early or late in the interview) and the effect of the problem solved just before were randomized. There were three exceptions to this random assignment. The two easiest problems were "Addition" and "Take Away Subtraction". One of these was the first problem the child was asked to solve. The children usually solved between five and ten problems during the first interview. Then at the beginning of the second interview the remaining "easy" problem was the first one the child received. The problem which was judged to be the most difficult, the multiple step problem with extraneous information, was always the last problem. If children were unable to complete the 15 problems in two interviews, the remaining problems were simply omitted. Thus, the multiple step--extraneous information problem was the one most frequently omitted. However, the other omissions were fairly evenly distributed among the twelve randomly ordered problems.

Problem solving tools and trial problems. During the problem solving sessions children were provided with calculators and popsicle sticks. They were also told that they could use drawings at any time. Before any of the experimental problems were administered children were asked to solve some trial exercises using these three tools. The subjects in the pilot study had been reluctant to use them so the trial exercises were incorporated into the interviews in the hope that with experience the students would feel freer to reach for a calculator or the sticks or make a drawing if needed.

Third and fourth grade children were given three trial problem pages. One page contained a verbal problem which children were told to solve using the sticks. The second page contained a verbal problem which was to be solved with a drawing and the third page contained a set of computational exercises for calculator solution. Fifth and sixth graders were asked to do the calculator exercises only. During the pilot study, the upper grades students seemed to be insulted when asked to solve problems with sticks or a drawing so the corresponding practice activities were omitted in their interviews.

Interview Methodology

Techniques for eliciting information. Prior to the pilot study we had hoped to gather information about how children made their choice of operation for solving a problem using "think aloud" techniques. That is, we anticipated that children would be able to give us information of the sort, "I think I'll divide because...." This method of collecting information was a total failure. First of all, the majority of children would not talk while solving a problem. Second, those who did, talked only about how they computed, not about how they made the choice of operation, for example, "I've gotta subtract . . . fifty-seven minus twenty-nine . . . I have to borrow a ten . . . seventeen minus nine is eight . . . " and so forth. We a'so tried questioning students after they solved the problem using questions such as, "Can you tell me why you divided to solve this problem?" Often the answers were meaningless. A typical meaningless answer was, "Cuz it says 'how much'." Other answers were logical, but clearly were not the basis for the child's choice of operation, as was the case for this answer: "Well you wouldn't subtract cuz the answer wouldn't be small enough, and

you wouldn't add because the answer would be too big and you wouldn't multiply because the answer would be even bigger. . . ." The child had chosen the correct operation for solving the problem immediately so it was highly unlikely that she had used this process of elimination in order to decide to use division. Finally, in response to the "Why did you divide?" question, many children would retell the entire story of the problem. . . . "It says you've got 350 books and a person can carry 15 books so divide." This kind of answer was informative in the sense that it told us that the child felt it was necessary to use "the whole problem" to determine an operation, but it still didn't give us any leads about how we could help less successful children solve problems.

Through trial and error we finally arrived at a method of interviewing that seemed to give us the information we were looking for, but instead of getting our most useful information from the successful problem solvers we obtained it from the less successful problem solvers.

A set of eight hints was developed for each problem. These hints were given whenever a student appeared to be unable to solve a problem. The procedures for using the hints in an interview are described in a later section. The hints were selected on the basis of teaching practices recommended in children's textbooks, in methods books and in professional journals. The hints were as follows:

a. Low verbal content

When using this hint, children were directed to "turn down the top of your paper and read what you see there." Printed on the back side of the paper was the same problem the child had just read, but restated in a less "wordy" format. The following is

an example of a low-verbal problem form:

> Need to move 350 books.
> Student can carry 15 books.
> How many students needed?

b. Smaller numbers

For this hint students were directed to "Fold up the bottom of your paper and read what you see there." The "smaller numbers problem" on the back side of the child's problem sheet was identical to the original problem except the number size had been decreased.

c. Action

When a child was given an "action hint" he was asked a question about whether the objects or quantities in the problem were being put together, removed, separated, compared or the like. For example the action hint for one of the measurement division problems asks the following questions:

> Is the librarian going to take away bunches of 15 books from the 350 books or is she putting bunches of books together? (blank for child's response) When you take away several bunches of 15, what should you do to solve the problem?" _____

For this particular hint, if the child answered the first question incorrectly or gave no response the interviewer did not ask the second question. The wording of each action hint was specific to the problem.

d. Part-part-whole

In subtraction and division problems the "whole" or total is given. Also, a "part" is given (subtraction and measurement division) or the number of parts is given (partition division). In the statement of addition and multiplication problems the parts are given,

and a total is sought. A typical "part-part-whole" hint, using the measurement division problem of the previous example, above, is

> "Are you given the total number of books or are you looking for the total?_____ Do you know the number of books in each part? _____ When you know the total and you know the number of books in each part of the total, what should you do to solve the problem?"

For this hint, as for the "action" hint, the interview discontinued the questioning whenever the child failed to give a correct response. The wording of part-part-whole hints was also specific to the problem.

e. Size of answer

The focus of this hint is the size of the answer in a very gross way. The largest number in the problem was picked and the child was asked if the answer was going to be larger or smaller than the designated number, for example, "Will the answer be larger or smaller than 350? _____ What could you do to get a smaller number? _____" This hint does not involve estimating even though on the surface it may appear that it does. If a child can actually estimate an answer, he already knows what operation to perform.

f. Question asks?

For this hint the interviewer said, simply, "What's the question asking for?" It was the same for all problems.

g. What's given?

Hint "g" was always the same regardless of the problem type. The interviewer asked, "What's given in the problem?" or some slight variation such as "What information does the problem tell you?"

h. Personalize

When the "personalize hint" was used, the interviewer retold the story of the problem using the child, his friends, his parents or his teacher as the problem personalities. The locale of the problem was also usually changed to a more familiar one. The numbers and the objects acted on in the problem remained the same as the original problem.

Hints for Rate Problems. Neither the "action" focus nor the "part-part-whole" focus are appropriate for rate problems, or are appropriate in only a very superficial way, so for rate problems a different type of hint was assigned to "c" and "d". The following are the "c" and "d" hints for rate problems.

c - Rate. Many-to-many correspondence

For this hint children were always directed to make a drawing, or in a few instances, a chart. A typical "c" hint was

> Give drawing hint, then: Try to show both the miles and the minutes on your drawing.

The phrase "many-to-many correspondence" was used to describe this hint because in all instances children were directed to focus their attention on both sets of objects in the problem. In the hint above it was miles and minutes. For a problem about the rate, 2 pencils for 15 cents, the child was directed to show both the pencils and the cents.

d - Rate. Unit rate

The focus of hint "d" was "rate per one." For example, if the problem said, "Joe can ride his bike 2 miles in 15

minutes," the child was asked, "Can you tell me how long it takes Joe to ride 1 mile?"

Summary of hints given in interviews. Hints a, b and h were problem solving "helps" which involved a transformation of the problems. Hints c, d, e, f, and g focused the child's attention on some specific attribute of the problem. The list below is a summary of the eight hints.

Problem Transformation Hints

a. Low verbal content

b. Smaller numbers

h. Personalize

Problem Attribute Hints

c. Action — c - Rate. Many-to-many correspondence

d. Part-part-whole — d - Rate. Unit rate

e. Size of answer

f. Question asks?

g. What's given

Construction of the Interview Recording Forms. A separate interview coding form was prepared for each problem. On the top of the form, the child's problem was replicated and the answer to the problem was given. All forms contained the following five sections:

ANSWER - In this section, the interviewer checked "Right" or "Wrong." Under each of these headings, however, a list was provided to give additional information. For example, under "Right" the interviewer could check "Wrong, but correct with interviewer hints."

MODE OF SOLUTION - The same five "modes" appeared on every form: paper and pencil, mental, calculator, manipulatives and drawings. More than one mode could be checked.

INTERPRETATION - With the exception of division problems the only information recorded here was whether or not the answer was labelled correctly (orally or in writing). For division problems involving remainders, the child's interpretation of the remainder was also recorded in this section.

OPERATION - The "operation" section contained a check list with the conventional operation for solving the problem listed first; then other operations including such procedures as "counting up" and "counting down" were listed. These additional operations and procedures were selected on the basis of behaviors which interviewers observed during the pilot study.

FOCUS - In the focus section of the recording form space was provided for writing a brief description of how the child solved the problem. During the pilot study, many check lists were tried for recording the child's problem solving strategies, but none could adequately account for the wide variation in children's behavior. This section also contained the list of eight hints described above which were written specifically for the problem. Finally a check list of mode (tool) hints appeared in this section. The mode hints were

____ Calculator

____ Sticks

____ Drawings

For rate problems and irrelevant data problems an additional section was provided on the interview coding form. These are described below.

RATE STRUCTURE - This section appeared only on the interview forms for the two rate problems. The check list for this section was

____ Appeared to use many-to-many correspondence

____ Did not use correspondence. Focused on arithmetic operations

____ Other: _____.

IRRELEVANT DATA - For both the single step and the multiple step irrelevant data problems the following check list was provided.

____ Ignored irrelevant data.

____ Attempted to use irrelevant data, but rejected.

____ Insisted on computation involving irrelevant data.

Sample copies of interview forms and the corresponding children's problem sheets are provided in Appendix B.

Conduct of the Interview. Prior to an interview, three of the eight focus hints were randomly assigned to each problem in the fifteen problem set. The hints were numbered 1, 2 and 3 corresponding to the order in which the interviewer was to give the hints. They were given, of course, only when the child needed help, so many sets of preassigned hints were never used.

During an interview, whenever a child appeared unable to make progress on a problem the interviewer began to give hints. If none of the three preassigned hints "worked," the interviewer was free to choose any hint not previously given that he or she thought might be helpful, or the interviewer could direct the child to use the sticks or a drawing. The use of the calculator on the other hand was suggested to the child at any time he appeared to be getting bogged down in the execution of an algorithm. The calculator was of no use to the child until he had decided on the operation for problem solution, so it was not an appropriate hint to give while he was still fumbling for a procedure. A flow chart of the interview technique appears on the next page. The calculator hint does not appear on the flow

Interview Flow Chart for a Single Problem

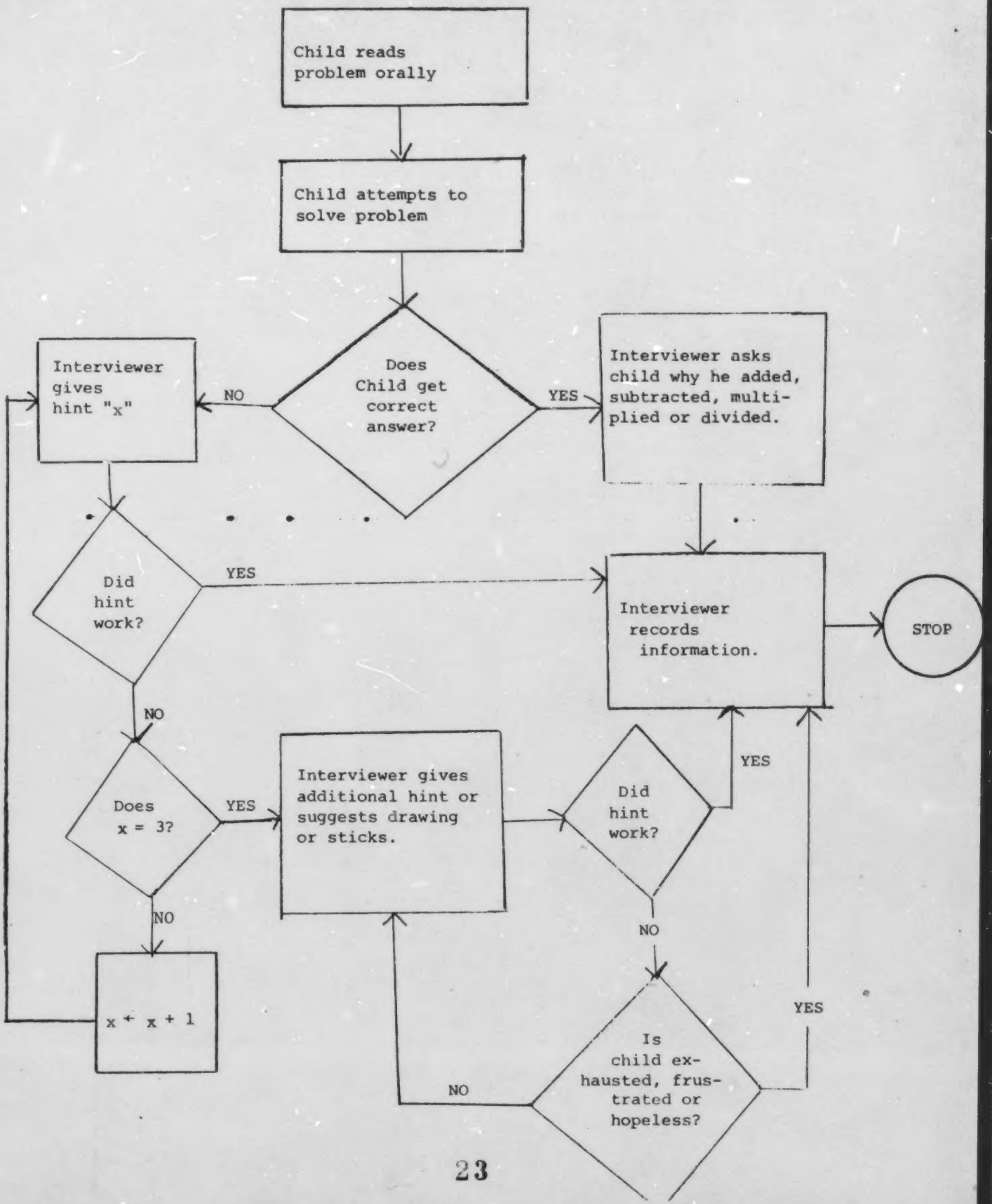

chart because it was suggested at any point in the interview where it was appropriate according to the "bogged down" criteria.

Three researchers conducted the interviews. Approximately 38 children were interviewed by the principal investigator. The remaining 124 children were interviewed by the two research assistants, Jean Geraghty and Jonathan Turner. Grade levels and ability levels were randomly distributed among the interviewers so that the effect of the interviewer's "style" has been controlled.

The Subjects

The subjects were selected from six elementary schools in Iowa City and four elementary schools in Cedar Rapids. High, average and low-average socio-economic levels were represented in the sample. The socio-economic level was based on the judgment of school officials. None of the schools could be considered low in terms of the socio-economic status of the neighborhoods from which the students were drawn. There are no schools in Iowa City which draw from a primarily low socio-economic level and the lowest socio-economic schools in Cedar Rapids were excluded because they were already involved in so many projects.

Six subjects were randomly selected from each of the four grades, three through six in each school, but with the stipulation that very low ability students should be excluded. Furthermore, the individuals who selected the students were asked to "adjust" the sample so that there was a balanced distribution of high, average and low ability students. Very low ability students were not selected because the purpose of the study was to determine how students who had the potential for solving problems could be helped to be better problem solvers.

The children who had been selected were given parent consent forms by their teacher and asked to return them in four days. We had hoped that by selecting six children we would have at least four children per grade level per school for whom parent consent for participation in the study would be granted. In several schools, consent was given by the parents of all six students. If the principal preferred that all students for whom parent consent had been given should be interviewed, we proceeded accordingly. In the other schools, only four children were interviewed, with the interviewer selecting the students based on their Iowa Tests of Basic Skills Mathematics Subtests scores.

For purposes of this study, ability levels were arbitrarily designated on the basis of percentile scores on the Mathematics Problem Solving Subtest of the ITBS as follows:

Low	20 to 62
Average	65 to 85
High	87 to 99

These percentiles are based on national norms. The only logic for the "split" at the sixty-second and eighty-fifth percentiles is that there was a break in the distributions at these points and it divided all four grade levels into groups of a reasonable size, with the "average" group larger than the other two for three of the four grade levels. The distribution of subjects by grade level and ability is given in Table 3.

Table 3

Number of Subjects by Grade Level

and by Ability Level

Grade	ITBS PROBLEM SOLVING SCORE			Total Number of Subjects
	Low 20 - 62	*Average* 65 - 85	*High* 87 - 99	
Third	14	13	13	40
Fourth	10	19	11	40
Fifth	14	18	12	44
Sixth	11	19	8	38
TOTAL	49	69	44	162

Results

Number of Problems Solved

Table 4 summarizes the results for the number of problems of each type solved correctly, incorrectly or omitted, combining the data for all four grade levels. For problems recorded in the column headed "correct without hints" children chose the correct operation(s) for solving the problem with no interviewer intervention. The interviewer in some instances, however, suggested the use of the calculator to complete some of the computation. Furthermore, if a child chose the correct process for problem solution, but made a minor error in computation, the datum was also recorded in this column. The same procedures for recording data, with respect to calculator usage and computational errors, were used for the column, "correct with hints."

The principal reason for inclusion of this table is to provide the reader with an overview of the data. The problems solved without hints will be the basis for establishing problem difficulty. The 613 problems solved with hints form the sample for the discussion in the section entitled "Effectiveness of Hints."

There were 2430 problems in all of which 2332 were attempted. It is important to note that of the 2332 attempted problems only 54 were recorded as "Wrong." That is, almost all students could solve most problems if they were guided to focus on certain attributes of a problem, if the problem were slightly transformed or if they were provided with an appropriate problem solving tool.

Problem Difficulty

In Tables 5 and 6 information on the difficulty of the fifteen problem types is examined. The data are reported in terms of percent of problems

Table 4

Number of Problems Solved

With and Without Hints

Problem Type	Correct		Wrong	Omitted
	Without Hints	With Hints		
Single Step Problems				
Addition	156	6	0	0
"Take Away" Subtraction	160	1	0	1
Comparative Subtraction	151	4	0	7
Additive Subtraction	145	13	0	4
Multiplication	146	12	0	4
Measurement Division No Remainder	126	28	3	5
Measurement Division With Remainder	109	42	5	6
Partition Division No Remainder	121	32	5	4
Partition Division With Remainder	113	39	3	7
Extraneous Information	114	45	1	2
Multiple Step Problems				
Using Multiplication	81	71	4	6
Using Division	74	70	12	6
Extraneous Information	50	58	19	35
Rate Problems				
Unit Rate A Whole Number	58	99	0	5
Unit Rate A Fractional Number	61	93	2	6
Totals	1665	613	54	98

solved without hints out of the problems attempted. The omitted problems were excluded from the analyses. In other words, the percentages recorded in Tables 5 and 6 were determined by the formula:

$$\text{Percent correct without hints} = \frac{\text{number solved without hints}}{\text{no. solved with and without hints} + \text{no. wrong}} \times 100$$

"Percent correct without hints" appeared to be a reasonable measure of problem difficulty since it is an indication of the number of problems students could solve independently.

Grade Level Analysis. Table 5 presents the results on problem difficulty by grade level. It is rather heartening to observe that the majority of the sixth graders in our sample performed very well on single step verbal problems. In fact even when the sixth grade was analyzed by ability level, the results were reassuring because average and low ability students as well as high ability students performed well. The lowest percentage of problems solved correctly without hints for any one problem type was 79% for the average ability sixth graders and 82% for the low ability students.

Reading across the table from third to sixth grade in the Single Step Problem category, there is, generally, the expected increase in performance with a few minor exceptions. Since the difficulty of the computation needed for solving the problems also increased from third through sixth grade, we have evidence to suggest that choosing the correct process for solving single step verbal problems increases with age irrespective of the computational difficulty.

The data for the single step "Extraneous Information" problems should be compared to the data for "Comparative Subtraction" and "Multiplication" problems, since the "s" form was of the former structure and the "u" form was of the latter structure. At each grade level the achievement on "Extraneous

Table 5

Percent of Each Problem Type Solved
Correctly Without Hints
by Grade Level

Problem Type	Third	Fourth	Fifth	Sixth	Total
Single Step Problems					
Addition	90	95	100	100	96
"Take away" Subtraction	98	100	100	100	99
Comparative Subtraction	97	95	100	97	97
Additive Subtraction	79	98	88	100	91
Multiplication	95	85	95	95	92
Measurement Division - No Remainder	66	82	86	87	81
Measurement Division - Remainder	46	56	86	89	70
Partition Division - No Remainder	58	67	88	92	77
Partition Division - Remainder	59	47	86	97	72
Extraneous Information	60	59	81	84	71
Multiple Step Problems					
Using Multiplication	62	46	45	55	52
Using Division	57	58	40	37	47
Extraneous Information	24	12	55	50	39
Rate Problems					
Unit Rate A Whole Number	29	16	37	58	37
Unit Rate A Fractional Number	21	18	42	66	39

Information" problems is lower than the corresponding problems without irrelevant data. This simply confirms prior research.

The results for Multiple Step Problems are far less encouraging than the results for Single Step Problems. First the achievement was considerably lower than for single step problems, but this was anticipated. Far more disconcerting is the fact that success in solving the multiple step problems in our study did not increase across grade levels. For the three problems, overall, achievement remained almost constant. This is particularly disturbing because of the similarity of the problems. The "Using Multiplication" problems and the "Using Division" problems were the same for both fifth and sixth graders. "Extraneous Information" problems for all four grades were identical in structure and wording except for number size. From third through sixth grade the computations for this problem were, in order of grade level, (4 x 10) - 24, (80 x 20) - 800, (80 x 20) - 800 and (75 x 25) - 1250. In other words, at least for fifth and sixth grades, problem difficulty did not increase by grade level, but neither did achievement.

The data in Table 5 were not adjusted for ability level. Since the number of students in each ability group differed from one grade level to the next there was concern that the apparent lack of improvement on Multiple Step Problems as grade level increased might be due to the variations in the sample. Therefore, for these three problems, Table 5A is provided showing the grade level by ability by problem type distributions.

Reading <u>across</u> Table 5A, for high ability students, one observes that there <u>was</u> an increase in performance on Multiple Step Problems as grade level increased. That is, for very high ability students (87th to 99th percentile),

Table 5A

Percent of Students by Ability Level and Grade Level
Solving Multiple Step Problems Correctly without Hints.

Problem Type	Ability	Third	Fourth	Fifth	Sixth
Using Multiplication	High	100	55	83	100
	Average	33	50	41	47
	Low	50	25	15	36
Using Division	High	54	82	50	63
	Average	50	44	44	32
	Low	67	56	23	27
Extraneous Information	High	30	22	75	100
	Average	29	8	53	47
	Low	13	0	36	18
Multiple Step Totals	High	64	55	69	88
	Average	37	38	46	42
	Low	47	33	24	27

the achievement on Multiple Step Problems increased with age. There was no corresponding increase, however, for the average (65th to 85th percentile) and low (20th to 63rd percentile) students. One would have hoped that average students would have shown some improvement, even if low students did not.

Referring again to Table 5, for "Rate Problems" there was a marked increase in performance from fourth to sixth grade with the pattern of achievement being quite similar for both problem types.

For grades three through five, rate problems were more difficult than multiple step problems but for sixth graders they were slightly easier than multiple step problems.

Ability Level Analysis. Table 6 summarizes the results on problem difficulty by ability level, summed over all four grades. With respect to Single Step Problems, there is very little difference in achievement between ability levels, with two exceptions, the "Measurement Division - No Remainder" and the "Extraneous Information" problems. In both cases the high ability students performed much better than the other two groups but the achievement of the average and the achievement of the low ability groups were about the same. The "Measurement Division" problem seems to be a fluke since similar differences were not observed for the other three division problems. Better performance on "Extraneous Information" problems by high ability students was expected.

For Multiple Step Problems, there is a high positive correlation between achievement and ability. This is also true for Rate Problems, but the differences between ability groups were not as great. It appears that for Multiple Step Problems, achievement is related primarily to ability level, but for Rate Problems, there is an interaction between ability and grade levels. For high ability students, the average score for Rate Problems (50%) is considerably below the average of the Multiple Step Problems (67%). For the other two groups the average score for Rate Problems is only slightly below the average of the Multiple Step Problem scores.

"Extraneous Information" problems in the Multiple Step category are more difficult than the other two problems for all three ability groups.

Effectiveness of Hints for Single Step and Multiple Step Problems

Because two of the hints for Rate Problems were different from the hints for the Single Step and Multiple Step problems and because the pattern of hint effectiveness was very different for Rate Problems than for the others, the results for Rate Problems will be reported separately.

Table 6

Percent of Each Problem Type Solved

Correctly Without Hints

by Ability Level

Problem Type	ITBS PROBLEM SOLVING SCORES		
	High 87 - 99	Average 65 - 85	Low 20 - 62
Single Step Problems			
Addition	98	97	94
"Take Away" Subtraction	100	100	98
Comparative Subtraction	100	95	98
Additive Subtraction	93	90	91
Multiplication	98	90	91
Measurement Division No Remainder	93	77	77
Measurement Division With Remainder	68	76	63
Partition Division No Remainder	79	75	77
Partition Division With Remainder	74	77	65
Extraneous Information	84	65	68
Multiple Step Problems			
Using Multiplication	84	45	33
Using Division	61	42	42
Extraneous Information	56	39	21
Rate Problems			
Unit Rate A Whole Number	40	40	26
Unit Rate A Fractional Number	59	35	26

Hints were provided for 473 Single Step and Multiple Step problems. This is the total of all problems solved with hints and all problems which were wrong. In all, 1078 hints were given during the course of the interviews. The number of times hints were used differed greatly among the eight focus hints and the two tool hints. The differences were due largely to the interview procedure which allowed the interviewer to select hints that "might work" when the list of three preassigned hints had been exhausted. Accurate records were not kept on "calculator hints." The number of times each hint (with the exception of the calculator hint) was given is listed in Table 7.

Table 7

Number of Times Each Type of Hint Was Used Excluding Hints for Rate Problems

Type of Hint	Number of Times Used
Focus Hints	
Low Verbal	110
Smaller Numbers	169
Action	158
Part-part-whole	117
Size of Answer	112
Question Asks?	110
What's Given?	95
Personalize	114
Tool Hints	
Manipulatives	23
Drawings	70
TOTAL	1078

The effectiveness of the eight focus hints and the two tool hints is examined in Tables 8 and 9. The percentages on these two tables were determined by dividing the number of times a hint "helped" by the number of times the hint was given.

The data for the "Personalize" hint has been separated from the rest of the data for two reasons. First, there was considerably less experimental control over the use of this hint. When an interviewer delivered this hint he or she had to create a story "on-the-spot." Several of the audio tapes on which the "Personalize" hint was used were reviewed and it appeared that the interviewer often did more "hinting" than just recreating the problem, so the data for this hint are contaminated by a variety of factors. Second, the usefulness of the hint as a teaching strategy is somewhat doubtful. It's not likely that children could be taught to "personalize" problems for themselves. The remaining seven focus hints have potential as strategies which children could be taught to use themselves or which could be incorporated into textbook materials.

Grade Level Analysis. The data reported in Table 8 for the effectiveness of hints by grade level is very confusing since few patterns are obvious from a cursory reading. "Personalize" appears to be a very effective hint for all grades. Even considering the fact that the results for this hint have been confounded by other factors, a conclusion that "personalize" is an effective help in problem solving seems warranted. Drawings are also a highly effective problem solving tool as are manipulatives. However, it should be noted that manipulatives were seldom used by children in any grade except the third grade, so the percentages reported for "manipulatives" in the fourth and fifth grades are based on a very small N.

Table 8

Summary by Grade Level of Percent of Times Hints Were Given
That They Helped Students Choose the Correct Process
For Solving All Problems Except Rate Problems

Type of Hint	Third	Fourth	Fifth	Sixth	All Grades
Focus Hints					
Low verbal content	30	19	29	17	24
Small numbers	32	57	35	60	46
Action	36	32	55	46	42
Part-part-whole	36	40	40	37	38
Size of Answer	29	38	43	32	35
Question asks?	14	4	10	28	13
What's given?	18	38	16	21	24
Tool hints					
Manipulatives	79	33	0	not used	70
Drawings	80	58	87	100	76
Other					
Personalize	54	42	56	42	49

The interpretation of the Focus Hints is not as clear cut as it is for the other two types of hints. In third grade the "Low Verbal," "Smaller Number," "Action," "Part-part-whole," and "Size of Answer" hints worked almost equally as well. The "Questions Asks?" hint and the "What's Given?" hint were far less useful. For both fourth and sixth graders the "Smaller Numbers" hint was a much more effective hint than the other six focus hints, but for fifth graders, the "Action" hint was superior to the rest of the hints. The "Action" hint, in fact, ranked among the top two most effective hints in three out of the four grades. The "Part-part-whole" hint was among the top three most effective hints for all grades. Transforming a problem into a low verbal form was the least effective problem solving aid for fourth and sixth graders; "Questions Asks?" was the least effective hint for third and fifth graders.

Ability Level Analysis. The analysis of hints by ability level is reported in Table 9. It turned out to be much more interesting and informative than the grade level analysis. The data reported in this table provided some surprises for the researchers. The results in Table 9 are probably the most useful information obtained from the entire study because they give us considerable insight into ways of helping children solve problems who are not highly successful problem solvers, the average and low ability students.

First of all it should be noted that very little information can be gained by reading across the table. Hints of all types generally worked better for high ability students than for the other two groups, so the percentages in the first column are higher overall than the other two. Reading down the columns to determine the relative effectiveness of the hints within ability groups provides the most interesting information. With respect to the Focus Hints, the within group rankings for the average

Table 9

Summary by Ability Level of Percent of Times Hints Were Given That They Helped Students Choose The Correct Process for Solving All Problem Types Except Rate Problems

Type of Hint	ITBS PROBLEM SOLVING SCORES		
	High 87 - 99	Average 65 - 85	Low 20 - 62
Focus Hints			
Low Verbal	47	23	16
Smaller Numbers	46	54	37
Action	56	48	32
Part-part-whole	35	41	36
Size of Answer	64	33	32
Question Asks?	33	8	13
What's Given?	32	29	13
Tool Hints			
Manipulatives	100	67	60
Drawing	67	73	81
Other			
Personalize	65	49	41

and low ability groups were quite similar, but the ranking of the effectiveness of focus hints for the high ability group is very different from the average and low. The rank of the focus hints from the most effective to the least effective for the high ability group is as follows:

1.	Size of Answer	64%
2.	Action	56%
3.	Low verbal	47%
4.	Smaller Numbers	46%
5.	Part-part-whole	35%
6.	Question Asks?	33%
7.	What's Given?	32%

The rankings of the focus hints for average and low ability students are presented side-by-side below to highlight the similarities between the two lists.

	Average			Low	
1.	Smaller Numbers	54%	1.	Smaller Numbers	37%
2.	Action	48%	2.	Part-part-whole	36%
3.	Part-part-whole	41%	3.	Action	32%
4.	Size of Answer	33%	4.	Size of Answer	32%
5.	What's Given?	29%	5.	Low verbal	16%
6.	Low verbal	23%	6.	What's Given?	13%
7.	Question Asks?	8%	7.	Question Asks?	13%

For youngsters who are most likely to need problem solving help, the average and low ability problem solvers, the "Smaller Numbers", "Action", and "Part-part-whole" hints appear to be the most useful aids. For low ability students, focusing on the size of the answer was almost equally as effective as the other three hints. The hints, "What does the question ask?" and "What information are you given in the problem?" are of doubtful value for all three

ability groups. It must be remembered that the "low group" for this study is not "low" in the usual sense (say 0 to 30th percentile on national norms). They are, generally, children who could be expected to profit from instruction in problem solving.

The "Low Verbal" hint was very ineffective for low ability students. It was somewhat more successful with average students, but still ranked sixth out of seven hints. The "Low Verbal" hint was, however, quite helpful for high ability students. The "Low Verbal" results were particularly interesting to the researchers. Low verbal content problems are used extensively in some of the popular contemporary children's mathematics textbook series and the usual argument for their inclusion is that they aid the less able problem solvers. For the children in our study this simply was not true. Why did this mini-problem format aid good problem solvers but not poor problem solvers? We think it's because the hint simply forced the child into a second reading of the problem which was sufficient in many instances for good problem solvers to grasp the problem, and that it probably would have been as effective for us to tell the child to re-read the original problem. In other words, it served to correct sloppy reading but it did not contribute to correcting more difficult comprehensional deficiencies.

Both of the tools, manipulatives and drawings, were highly effective problem solving "helps," as was the "Personalize" hint for all ability groups.

Effectiveness of Hints for Rate Problems

Table 10 presents a summary of the effectiveness of hints for the two rate problems by problem type. These problems were also analyzed by ability level and by grade level, but the patterns of hint effectiveness were so similar to the information provided in Table 10 that the analyses are not

Table 10

Percent of Hints Given Which Helped Students in Grades Three Through Six Solve Rate Problems

Type of Hint	Problem Type	
	Unit Rate a Whole Number	Unit Rate a Fractional Number
Focus Hints		
Low Verbal	20	17
Small Numbers	8	17
Many-to-many-Correspondence with Drawing	98	92
Find the Unit Rate	80	36
Size of Answer	0	4
Question Asks?	9	0
What's Given?	30	4
Other		
Personalize	23	25

included. The Tool Hints section has been omitted from this table. The "Many-to-many Correspondence" hint involved using a drawing and very few children used manipulatives, so there was no useful information on "tools" apart from the hint in which a drawing was used.

As the reader reviews Table 10 it becomes quite obvious why the data for these two problems were analyzed separately. None of the hints, not even the "Personalize" hint, worked very well except the "Many-to-many Correspondence" hint and the "Unit Rate" hint. Both of these hints were very effective when used with the Unit Rate a Whole Number problem. The "Unit Rate" hint was not very helpful, however, with problems in which the unit rate was a fractional number. Recall that this latter problem was of the sort, "2 for 15" where the corresponding unit rate, 1 for 7½, involved the fraction ½. In the grade level analysis it was observed that even for sixth graders who should have reasonable proficiency working with fractional numbers, the "Unit Rate" hint was not particularly useful for this latter problem type.

The "What's Given?" hint was moderately useful for the Whole Number problem. We observed that in responding to this hint children typically reread the problem and in so doing some of them noted that the problem said for example," THREE pencils for 15 cents." Prior to this rereading, the child had been processing the information as if the price had been one pencil for 15 cents. Most children however would tell the interviewer, "It says three pencils for 15 cents," and still were not able to deal with the information.

Calculator Usage

Calculator usage is examined in Tables 11 and 12. The information on these tables is biased by the fact that interviewers encouraged children

to use the calculator whenever they were having difficulty with computation. Although we attempted to keep a record of "calculator hints" it was not accurate. Since the question of calculator usage was not one of major interest in the study, we did not want to take the great amount of time involved in rechecking the coding forms against the tapes of the 324 interviews. Our records on whether or not calculators were used are accurate, but we are not sure when the calculator usage was initiated by the interviewer and when it was initiated by the student. The data are presented, however, so that other researchers who are interested in the relationship of the calculator to problem solving might note some trends which warrant more careful study.

Grade Level Analysis. Table 11 shows the amount of calculator usage for each of the four grade levels. Only problems which were solved correctly were examined in this analysis, "Correctly" means, in this context, without hints and with hints.

The highest calculator usage on Addition and Subtraction problems occurred at the third grade. The highest calculator usage on Multiplication and Division problems occurred at the fourth grade. Both of these results have fairly obvious explanations. In the third grade, the experimental multiplication and division problems required computation with basic facts only, but the addition and subtraction problems involved computation with two digit numbers. Third graders are less competent with the addition and subtraction algorithms than students in other grades so their frequent use of calculators for these algorithms is understandable. The fourth grade multiplication and division problems required the use of the algorithms, hence the extensive use of the calculator at this grade level is explained by the fact that fourth grade is the year during which the algorithms are

Table 11

Percent of Problems Solved Correctly Which Were Solved With the Aid of a Calculator by Grade Level

Problem Type	Third	Fourth	Fifth	Sixth
Single Step Problems				
Addition	45	38	36	29
"Take Away" Subtraction	53	46	27	24
Comparative Subtraction	50	34	30	24
Additive Subtraction	51	30	37	24
Multiplication	29	60	36	42
Measurement Division No Remainder	54	83	65	61
Measurement Division Remainder	50	86	48	34
Partition Division No Remainder	58	89	67	47
Partition Division Remainder	39	67	50	29
Extraneous Information	49	38	37	24
Multiple Step Problems				
Using Multiplication	62	74	54	34
Using Division	60	71	59	61
Extraneous Information	33	71	54	40
Rate Problem				
Unit Rate A Whole Number	13	16	5	5
Unit Rate A Fractional Number	31	26	23	16
Total	45	54	42	33

taught. From fourth through sixth grades calculator usage by our subjects decreased as students became more proficient computers.

Ability Level Analysis. The ability level analysis for calculator usage is presented in Table 12. The table indicates that high ability students used the calculator less on every problem type than the other two ability groups. This was expected. Calculator usage by average students was very similar to the calculator usage of low students. All ability groups used the calculator more frequently for division problems than for problems requiring the other three operations.

Mode of Solution by Ability Level and by Problem Solving Independence

For fifth and sixth grade students there was a high positive correlation between problem solving performance on the fifteen experimental problems and the ITBS problem solving subtest scores. For third and fourth grades, however, the differences between ability groups on the experimental problems were not as distinct as the ITBS scores would seem to predict. At the third grade level, the percent of problems solved correctly without hints for the high, average and low groups respectively were 70%, 63% and 63%. For the fourth grade the range of the scores was somewhat greater, but the differences between ability groups were still not as great as at other grade levels. The percent of problems solved correctly without hints for fourth graders were, in order of decreasing ability, 76%, 67% and 64%.

Because at the third and fourth grade levels the range of the ability groups' average scores on the experimental problems was so narrow we decided to investigate the effect of the interview methods on the scores, in particular, the effect of the availability of manipulatives, drawings and calculators. Tables 13 and 14 were developed for this purpose. The percentages appearing on the tables are the percent of students within the

Table 12

Percent of Problems Solved Correctly Which Were Solved with the Aid of a Calculator by Ability Level

Problem Type	ITBS PROBLEM SOLVING SCORES		
	High 87 - 99	Average 65 - 85	Low 20 - 62
Single Step Problems			
Addition	32	38	41
"Take Away" Subtraction	27	43	39
Comparative Subtraction	26	35	41
Additive Subtraction	19	43	40
Multiplication	30	48	43
Measurement Division No Remainder	60	70	63
Measurement Division With Remainder	43	64	51
Partition Division No Remainder	51	75	65
Partition Division With Remainder	36	54	44
Extraneous Information	25	40	45
Multiple Step Problems			
Using Multiplication	41	56	68
Using Division	40	79	60
Extraneous Information	36	49	60
Rate Problems			
Unit Rate A Whole Number	5	10	13
Unit Rate A Fractional Number	23	30	26
Total	33	48	46

Table 13

Third Grade: Mode of Solution by Ability Level by Problem Solving Independence

Problem Solving Independence		Paper and Pencil Exclusively or Paper and Pencil with Mental	Mental Exclusively	Drawing or Manipulatives Alone or in Combination with Other Modes	Calculator Alone or in Combination with Other Modes
Without Hints	H	48 (26%)	16 (9%)	14 (8%)	52 (28%)
	A	22 (13%)	18 (11%)	13 (8%)	61 (36%)
	L	19 (10%)	23 (12%)	11 (6%)	71 (37%)
With Hints	H	10 (5%)	4 (2%)	24 (13%)	18 (10%)
	A	3 (2%)	5 (3%)	35 (20%)	24 (14%)
	L	7 (4%)	9 (5%)	36 (19%)	21 (11%)

Table 14

Fourth Grade: Mode of Solution by Ability Level by Problem Solving Independence

Problem Solving Independence		Paper and Pencil Exclusively or Paper and Pencil with Mental	Mental Exclusively	Drawing or Manipulatives Alone or in Combination with Other Modes	Calculator Alone or in Combination with Other Modes
Without Hints	H	56 (36%)	13 (8%)	2 (1%)	48 (31%)
	A	60 (23%)	3 (1%)	7 (3%)	101 (39%)
	L	24 (19%)	4 (3%)	5 (4%)	48 (38%)
With Hints	H	7 (4%)	2 (1%)	15 (10%)	18 (12%)
	A	16 (6%)	3 (1%)	25 (10%)	47 (18%)
	L	3 (2%)	4 (3%)	22 (17%)	28 (22%)

ability group who fell into the indicated cell. For example, in the first cell in the first column of Table 13 H - 26% means "26% of the problems solved correctly without hints by high students were solved using paper and pencil exclusively." The total percent for each ability group is slightly higher than 100% because some students used both drawings and a calculator and have, therefore, been recorded twice.

On both tables the first cell in the first column, "Paper and Pencil-Without Hints", is interesting because the range of the ability groups' scores within this cell is much greater than the range of the scores overall. That is, the percent of each ability group who solved the experimental problems correctly using only paper and pencil corresponds much more closely to the ITBS scores than do the overall percentages.

Examining the "without hints" rows, in both third and fourth grade it appears that the differences between ability groups on the experimental problems were narrowed because of calculator usage. The use of drawings and manipulatives did not contribute to narrowing the differences at the third grade level, and only slightly at the fourth grade level. In the third grade, the percent of problems solved by manipulatives or drawings was about the same for the three ability levels.

With respect to mental solutions, again looking at the "without hints" rows, very few problems were solved mentally by fourth graders which is understandable since the numbers in the problems were two and three digit numbers. It's interesting that in the third grade the number of mental solutions increased as ability level decreased. However, the differences are too small to be considered significant.

For all ability levels, students who needed help solving problems (the "with hints" rows on the tables) used drawings or manipulative and calculators extensively as compared to paper and pencil solutions. Of these students,

those who were third graders used drawings and manipulatives somewhat more frequently than calculators. At the fourth grade the trend was reversed.

Analyses for Selected Problem Types

Additive Subtraction Problems. Additive subtraction problems are problems of the sort, "Jane has 27 cents. How much more money does she need to buy a Star Wars poster that costs 49 cents?" A quantity is given and the student is asked how much more has to be put with it. If one were to describe the above with an equation, $27 + x = 49$ would be an appropriate expression of the problem relationships. Some authors refer to this problem type as "joining" problems. Gibb (1956) and Carpenter (1979) have both reported that first graders use "adding on" and "counting up" strategies for solving additive subtraction problems, so we were interested in whether or not this strategy persisted in older children, particularly when the numbers were larger than the numbers used in the other studies.

Thirty-seven third graders in all solved the experimental additive subtraction problems correctly. Twenty-six solved the problems using subtraction. The remaining 11 problems were solved using adding on or counting up techniques. This is 30% of all problems solved so it's reasonable to conclude that the strategies are still used by third graders, but less extensively than by younger children.

The computation for the continuous form of the additive subtraction problem was 25 - 16. For the discrete form the computation was 49 - 35.

The analysis for the 11 problems solved by adding on and counting up is presented in Table 15. The C's and D's on the table represent continuous and discrete problems. The adding on technique, which would seem to be

Table 15

Methods Other than Subtraction Used by Third Graders for Solving Additive Subtraction Problem

Level of Independence	Ability Group	Counting Up	Adding On	Totals
Without Hints	High	3 (2C,1D)		3
	Average	3 (1C,2D)		3
	Low	1 (C)	1 (C)	2
With Hints	High	1 (D)		1
	Average			
	Low		2 (D)	2
TOTALS		8	3	11

the more sophisticated of the two procedures was used only by low students, but the counting up technique was evenly distributed among high, average and low students. There was an equal distribution of continuous and discrete problems solved by counting up so this variable seems to have little effect on counting up usage. Although all three problems solved by adding on were discrete quantities problems the number is so small that no conclusion can be made about the relationship of adding on and the discrete-continuous factor. Most students who used counting up did not have help from the interviewer.

The additive subtraction problems for fourth grade were also analyzed. All but four of the fourth graders used subtraction for the solution. Three used counting up and only one used adding on. Two of the students who did not use subtraction were average students and two were low ability students.

Students who counted up invariably used the popsicle sticks. Those who added on, however, had varied techniques. One student who added on estimated. For the problem having the structure $35 + x = 49$, she added 13 to 35 on the calculator, got 48, and then tried 14 (which was correct). Others used adding on tens, as for example "35 and 10 is 45 and 4 is 49 so 14." For the problem having the form $75 + x = 98$, another "adding on" student said, "From 75 to 85 is 10 miles. From 85 to 90 is 5 miles. From 90 to 98 is 8 miles so 23" (adding 10 + 5 + 8).

It was of interest to the researchers that only three students out of all third and fourth graders used the adding tens "adding on" strategy and all three of these students were low ability students. Yet adding on tens is a very good mental computation procedure. One wonders why only low students used "good" mental computation techniques.

Unlike and Similar Extraneous Data. The Single Step "Extraneous Information" problem was administered in two forms. The 's' form contained extraneous information similar to the problem data; the 'u' form contained extraneous information which was unlike the problem data. The results are confounded by the use of different problem structures for the two data factors, but are interesting nevertheless. The problems which had "unlike" extraneous information were multiplication problems. Comparative subtraction was used in the problems with "similar" extraneous information. Although the problems are provided in the appendices, they are repeated here so the reader can have them in view while examining the results.

	Similar Data	*Unlike Data*
-Third Grade-	There are 24 people in line at a movie theater. In the theater there are 18 empty seats. There are 6 seats in each row. How many people will not get a seat? Ans. 6 people	A 6 ounce can of frozen lemonade costs 20¢. If it takes 4 cans to make enough lemonade for my picnic jug, how much does it cost to fill my jug with lemonade? Ans. 80¢
-Fourth Grade-	There are 480 people in line at a movie theater. The theater seats 456 people, with 24 seats in each row. How many people will not get a seat? Ans. 24 people	A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade to fill my picnic jug, how much does it cost to fill my jug with lemonade? Ans. $1.92
-Fifth Grade-	There are 1125 people in line at a movie theater. The theater seats 975 people with 25 seats in each row. How many people will not get a seat? Ans. 150 people	A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade to fill my picnic jug, how much does it cost to fill my jug with lemonade? Ans. $1.92

	Similar Data	*Unlike Data*
-Sixth Grade-	There are 1125 people in line at a movie theater. The theater seats 975 people with 25 seats in each row. How many people will not get a seat? Ans. 150 people	A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade to fill my picnic jug, how much does it cost to fill my jug with lemonade? Ans. $1.92

On Table 16, the results for the u-s factor by grade level are given. The difference between achievement on the two types of extraneous data problems is greatest for fourth grade, where the unlike data problem was much more difficult than the similar data problem. The unlike data problem is a multiplication problem and multiplication is difficult for fourth graders, but it is unlikely that all of the difference is attributable to the problem structure. Referring back to Table 5, for fourth graders, the difference between success on Multiplication problems and Comparative Subtraction Problems was 10% while the corresponding difference on Table 16 is 49%. For the other three grade levels, the similar data problem was slightly more difficult for third graders, for fifth grade there was almost no difference and for sixth graders the unlike data problem was more difficult.

Table 16

Percent of Unlike and Similar Data Extraneous Information Problems Solved Correctly Without Hints for Third Through Sixth Grades

Problem Type	Third	Fourth	Fifth	Sixth	All Grades
Unlike (Multiplication)	65	39	81	79	65
Similar (Comparative Subtraction)	57	88	82	89	78

The results by ability level shown on Table 17 are far more consistent than the grade level results. For all ability levels, the unlike data problem was more difficult. It is also true, however, that for all ability groups multiplication problems were more difficult than comparative subtraction problems. The differences between the two problem types (computed from the Table 6 data) were 2% for high ability students, 5% for average students and 7% for low students. For each group the differences on Table 17 between the unlike and similar problems are greater than these, but certainly not great enough to conclude that unlike data problems are more difficult than similar data problems. On the other hand, prior research has shown that similar data problems are more difficult than unlike data problems. This conclusion is not supported by our results.

Table 17

Percent of Unlike and Similar Data Extraneous Information Problems Solved Correctly Without Hints for Each Ability Level

Problem Type	High	Average	Low
Unlike (Multiplication)	79	58	63
Similar (Comparative Subtraction)	88	71	76

Rate Problems. For purposes of this paper, rate problems are defined to be those in which a many-to-many correspondence or a many-to-one correspondence is implied, for example:

(a) If Mary can ride her bicycle 2 miles in 20 minutes, how long will it take her to ride 8 miles?

and (b) If 3 ping pong balls cost 85¢, how much will 12 ping pong balls cost?

Problem (a) indicates a correspondence between miles and minutes; 2 miles to 20 minutes. In problem (b) the given correspondence is 3 balls to 85 cents.

Two strategies of solving these problems were identified. In the strategy which we have called "Unit Rate" the student first finds the cost per one, time per one and so forth, and then multiplies to determine the cost, time or whatever for the total number of units. A typical "unit rate" strategy for problem (a) would be the following:

$20 \div 2 = 10$, so 10 minutes for 1 mile;

$8 \times 10 = 80$, so 80 minutes for 8 miles.

In problem (b) the unit rate is a fractional number. The computation using the "unit rate strategy" for (b) might be similar to this.

$85 \div 3 = 28\ 1/3$ so $28\ 1/3$ cents for each ball;

$12 \times 28\ 1/3 = 340$, so \$3.40 for 12 balls.

Using the many-to-many correspondence method of problem solving the student does not compute the unit rate. The child's solution is based on this kind of thinking pattern (for problem (a)).

2 miles for 20 minutes;
2 miles for 20 minutes;
2 miles for 20 minutes;
2 miles for 20 minutes.
That's 8 miles in 80 minutes.

For problem (b) the child's thinking, using the many-to-many correspondence strategy would be similar to the following:

3 balls for 85¢;
3 balls for 85¢;
3 balls for 85¢;
3 balls for 85¢.
4 x .85 = $3.40 so 12 balls for $3.40

There are of course many variations of the descriptions above. Furthermore some children mixed the two strategies. One of the problems for which children mixed the two strategies was:

Mary can buy 3 sticks of candy for 15¢.
How much will 7 sticks cost?

You will notice that in this problem, the total number of units desired, is not a multiple of 3. Quite a few children solved the candy problem using reasoning of the sort:

/ / / / / / / 15 ÷ 3 = 5, so the last stick of candy costs 5¢. 15 + 15 + 5 = 35.
15¢ 15¢

In a prior section of this paper entitled "Effectiveness of Hints for Rate Problems" we found that focusing children's attention on the unit rate helped them solve problems of type (a), but was not very useful in solving problems of type (b) where the unit rate is a fractional number. The many-to-many correspondence hint worked well with both types of problems. These results, however, dealt only with those children who needed help solving rate problems. In Table 18 we show the strategies used by children who were able to solve problems without interviewer assistance. The percentages recorded in this table are based only on the number of students who solved the problem without hints so the rows in each case total 100%, except for slight variations due to rounding. One

Table 18

Strategies Used by Students at Each Grade Level Who Solved Rate Problems Independently

Problem Type	Grade	Many-to-Many Correspondence	Unit Rate	Mixed
Unit Rate A Whole Number	Third	57% (8)	21% (3)	21% (3)
	Fourth	67% (4)	33% (2)	0% (0)
	Fifth	25% (4)	63% (10)	13% (2)
	Sixth	50% (11)	50% (11)	0% (0)
	Totals	47% (27)	45% (26)	9% (5)
Unit Rate A Fractional Number	Third	100% (11)	0% (0)	0% (0)
	Fourth	100% (7)	0% (0)	0% (0)
	Fifth	78% (14)	17% (3)	0% (0)
	Sixth	92% (23)	8% (2)	0% (0)
	Totals	90% (55)	8% (5)	(0)

problem which was correctly solved by a fifth grader is not accounted for in the table because it could not be categorized as belonging to any of the three strategies.

We were surprised to find that the pattern of usage of the many-to-many correspondence strategy and the unit rate strategy displayed in Table 17 was very similar to that for children who were given hints when solving rate problems. The unit rate method was used almost as often as the many-to-many correspondence method when solving problems having a whole number unit rate. But for those problems having a unit rate which involved a fraction, children used many-to-many correspondence more than nine times as often as the rate per one method. Fifth graders solving "Unit Rate a Whole Number" problems were the only exception. They used the unit rate method much more frequently than the many-to-many correspondence procedure when solving these problems. We can only speculate that since division is emphasized in fifth grade they may have had recent instruction in using division to find unit prices.

Methods of solution used by students who solved problems independently were also analyzed within ability levels. The relative usage of the strategies was about the same for all ability levels and was very similar to the percentages for "total" given in Table 17, so the data for ability levels have been omitted.

Both the "Unit Rate a Whole Number" and "Unit Rate a Fractional Number" problems were administered in the continuous and discrete forms. Table 19 shows the effect of the continuous-discrete factor on method of solution for all students who solved the problems correctly (without or with hints).

Table 19

The Effect of the Continuous-Discrete Factor on
The Strategies Used to Solve Rate Problems

Type of Quantity	Level of Independence	N	Many-to-Many Correspondence	Unit Rate	Mixed
Unit Rate A Whole Number					
Continuous	Without Hints	24	19%	11%	0%
	With Hints	57	53%	16%	1%
	Totals	81	72%	27%	1%
Discrete	Without Hints	34	14%	22%	8%
	With Hints	42	25%	25%	5%
	Totals	76	39%	47%	13%
Unit Rate A Fractional Number					
Continuous	Without Hints	25	31%	1%	
	With Hints	52	57%	10%	
	Totals	77	88%	12%	
Discrete	Without Hints	36	40%	5%	
	With Hints	41	47%	6%	
	Totals	77	87%	12%	

For "Unit Rate a Fractional Number" (the lower part of the table) it seems reasonable to conclude that the continuous-discrete factor had no impact on the solution strategy. Well over 80% of both the continuous and the discrete problems were solved using many-to-many correspondence methods, and the over-whelming reliance on many-to-many correspondence was evident for both the continuous and discrete factors whether or not children needed hints to solve the problems.

The "Unit Rate a Whole Number" portion of the table, however, tells quite a different story. Discrete problems were solved about the same number of times with both strategies. Continuous problems on the other hand were solved far more frequently by the many-to-many correspondence method. Furthermore these patterns were fairly constant within the "Without Hints" and "With Hints" groups.

It is not at all clear to the researchers why the continuous-discrete factor interacted with the strategy variable for the one problem type but not for the other.

Some selected examples of children's solutions are provided below to illustrate the uses of the strategies.

Problem Type: Unit Rate a Fractional Number - Continuous Quantities

Grade Level: 3

> If Joe can ride his bike 2 miles in 15 minutes, how long will it take him to ride 8 miles?

Paul, a high ability student solved the problem immediately. He picked up the calculator and entered 15 + 15 + 15 + 15 = . When asked why he added four 15's he said, "two, four, six, eight . . . " This was recorded as a many-to-many correspondence strategy.

Problem Type: Unit Rate a Fractional Number - Discrete Quantities

Grade Level: 5

> The cost of 3 ping pong balls is 85¢. How much will 12 ping pong balls cost?

Sara, a student whose ITBS problem solving score was on the lower fringes of the average group also solved her problem without interviewer assistance. She explained after she had solved the problem, "There are 3 balls in each package and 12 balls would make 4 packages and each pack costs 85¢ so multiply four times eighty-five to get $3.40." It's rather interesting

that this child invented "packages" to help her solve the problem. This was also recorded as a many-to-many correspondence solution strategy, because she never considered the price of <u>one</u> ping pong ball. She used the idea, "a package for 85¢", in other words "three for 85" throughout the problem solution.

Chris a high ability sixth grader while solving the same problem had some difficulty performing the computation but he chose the correct processes for solution immediately. First he divided: $3\overline{)85}$ getting 28 1/3. He then tried: 28 1/3 x 11 3/3 (an obvious mix-up about recent instruction on multiplication and subtraction of fractional numbers) and as might be expected, had trouble. The interviewer then suggested that he use the calculator and also told him to use 28.333 for 28 1/3. Finally he got 339.996 on the calculator and interpreted the result correctly as 340. At the conclusion he said, "Whew, there <u>must</u> be an easier way!" The strategy recorded for this solution was "Unit Rate."

Jennifer, an average ability third grader used a drawing, <u>without</u> a suggestion from the interviewer, to solve her problem:

Problem Type: Unit Rate a Fractional Number - Discrete Units

> The costs of 2 pencils is 15¢. How much will 6 pencils cost?

Jennifer's solution is shown in figure 1. It is obviously a many-to-many correspondence solution.

The cost of 2 pencils is 15¢. How much will 6 pencils cost? 45

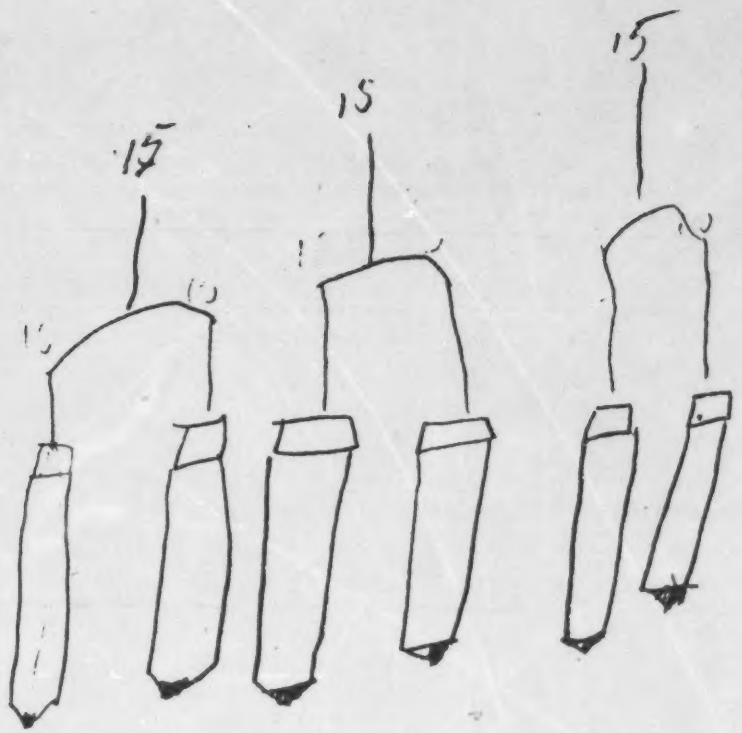

Figure 1. Jennifer's rate problem solution.

SUMMARY OF FINDINGS

Problem Difficulty --

Single Step Problems

1. By sixth grade most single step problems could be solved by children of all ability levels.

2. Children's performance on single step problems improved each year from third through sixth grades.

3. Single step problems with extraneous information were more difficult than problems having the same structure without the extraneous information.

4. With the exception of extraneous information problems, there was very little difference in achievement between ability levels on single step problems.

5. High ability students performed much better than the other two groups on extraneous information problems. The achievement of average and low ability students was about the same.

Multiple Step Problems

6. Success in solving multiple-step problems did not increase across grade levels.

7. There was a marked increase in performance from low to average and from average to higher ability students on multiple step problems.

8. The multiple step problem with extraneous information was much more difficult for every ability group than the other multiple step problems.

Rate Problems

9. For grades three, four and five rate problems were more difficult than multiple step problems, but for sixth graders they were slightly easier than multiple step problems.

10. There was a decided increase in achievement from fourth to sixth grades on rate problems. Third graders, however, achieved slightly better than fourth graders.

11. The performance of high ability students on rate problems was considerably below their performance on multiple step problems.

Effectiveness of Hint --

Single Step and Multiple Step Problems

12. For all grade levels and all ability levels, "Personalize" was a helpful problem solving hint.

13. For all grade levels and all ability levels, manipulatives and drawings helped children solve problems.

14. For all grade levels and all ability groups decreasing the size of the numbers was very effective in helping students solve problems.

15. Focusing the child's attention on the action implied in a problem, on the size of the answer, and on the part-part-whole relationships were all useful techniques for helping children solve verbal problems, however, the action hint was the only one which was among the top three most effective hints for all ability groups.

16. Focusing the child's attention on the information given and on the question asked in the problem were, overall, the least successful procedures for assisting children in the selection of the correct operation for problem solution.

17. Restatement of a problem in low verbal form was very ineffective for low ability students, somewhat more effective for average students although less helpful than most other hints. The low verbal form was quite helpful for high ability students.

Rate Problems

18. Focusing a child's attention on the many-to-many correspondence in a rate problem, using a drawing, was a highly effective strategy for both the "Unit Rate a Whole Number" and "Unit Rate a Fractional Number" problems.

19. Focusing the child's attention on the "rate per one" helped children solve the "Unit Rate a Whole Number" problem but was considerably less effective for the "Unit Rate a Fractional Number" problem.

20. None of the other hints, Personalize, Smaller Numbers, Low Verbal Form, Size of Answer, "Question asks?", and "What's given?", was very useful in helping children solve rate problems.

Calculator Usage --

21. The pattern of calculator usage by average students was very similar to the calculator usage of low ability students. Both groups used a calculator more often than high ability students.

22. Fourth graders used calculators much more extensively than any other grade level.

23. Calculators were used more often on division problems than any other problem type.

24. It appears that differences in problem solving achievement between ability groups in third and fourth grade were smaller than anticipated because of extensive calculator usage, not because of manipulatives and drawings.

Special Analyses --

Additive Subtraction Problems

25. "Counting up" and "adding on", procedures commonly used by first graders to solve additive subtraction problems, continue to be used by a substantial number of third graders, but by fourth grade, counting up and adding on were used by very few students.

Unlike and Similar Extraneous Data

26. Contrary to the results of prior research, problems containing extraneous information which was similar to the problem data were not more difficult than problems with "unlike" data.

Rate Problems

27. Students who solved rate problems without interviewer assistance used the many-to-many correspondence solution strategy about as frequently as the unit rate strategy on problems in which the unit rate was a whole number.

28. Independent problem solvers used the many-to-many correspondence strategy more than nine times as often as the unit rate strategy when solving problems having a fractional number unit rate.

29. There was an interaction between the continuous-discrete variable and solution strategy, but only for "Unit Rate a Whole Number" problems.

IMPLICATIONS FOR INSTRUCTION

Areas of Concern

Elementary school students can solve single step verbal problems. In fact, by the time they are in sixth grade even fairly low ability students achieve well on single step problems. The issue of multiple step problems is quite another matter. As expected, achievement on multiple step problems was lower than for single step problems. The more disturbing fact about multiple step problems, however, is that there appears to be almost no improvement from third to sixth grade. It is not known whether this is because children do not have enough practice on solving multiple step problems, or because the opportunity to solve multiple step problems is provided, but the methods of instruction are ineffective. Both factors should be present in the elementary school mathematics program, opportunity and appropriate instruction.

A survey of five popular sixth grade mathematics textbooks was conducted recently by the principal investigator to determine trends in programs for teaching problem solving. It was found that the number of multiple step problems appearing in the books varied greatly from approximately 17 to 126, with a median of 35. This means, obviously, that three out of the five textbooks contained 35 or fewer multiple step problems. For children using these series opportunity is limited. In the textbook series containing the greatest number of multi-step problems the only advice given for teaching problem solving was to encourage children to "think it through." In this latter series, opportunity is provided, but appropriate instruction is not.

There is, of course, one other possible explanation for the lack of improved achievement from one grade level to the next. That is that

elementary school students are incapable of learning to solve multiple step problems. The teaching profession should be unwilling to accept this hypothesis until the effect of better methods of instruction and the effect of more opportunity to solve multiple step problems are investigated.

Rate problems are another area of concern. The rate problems used in this study were very easy problems at all grade levels. Yet even for sixth graders who performed considerably better than younger children the average percent of rate problems which were solved without interviewer assistance was 62%. The rate concept is a very powerful one, prevading a wide variety of applications of mathematics: conversion of measurements - 4 quarts per one gallon, 100 centimeters per one meter, 100° between the freezing point and the boiling point on the Celsius scale for 180° on the Fahrenheit scale; consumer problems - 12 ounces for 39¢, 2 cans for 95¢, 54.6¢ per half gallon; scale problems - 1/4" to 1 foot, 1" to 10 miles and so on. There is some evidence that children as young as fourth graders can be taught to solve rate problems (Zweng, 1960), but additional support is needed for the conjecture. Producers of children's textbooks should be made aware of the deficiency of their programs with respect to children's achievement on this very useful type of problem.

Modern textbooks give very little attention to dealing with extraneous information in verbal problems. Even if they did, it's highly likely that extraneous information problems would still be more difficult than problems without the "useless" data. Extraneous data problems are, nevertheless, a third area of concern.

Instructional Methods

Compared to other problem solving "helps" used in this study, "personalize" was highly successful, however, it probably has very little promise as an instructional method. If all else fails, a teacher might

help a student by "personalizing" but to rely on personalizing as the principal methodology seems inadvisable. It is a highly teacher dependent technique. It's very unlikely that children who are poor problem solvers could learn to personalize problems for themselves.

Many other useful "helps" for problem solvers were found which have more potential for instruction than personalize. Focusing the child's attention on the action implied in a problem, on the part-part-whole relationships and on the size of the answer were all good techniques. It remains to be seen, however, whether or not children can be taught to use these strategies on their own.

Teaching children to make drawings also holds promise. Drawings were not only an effective hint, but we observed that for some children, once they had had success solving a problem with a drawing they appeared to use drawings more frequently thereafter. Whether or not children can be taught to make drawings independently of teacher direction and encouragement is still an open question. A study by Nelson (1974) showed very little improvement as the result of drawing instruction except for some very special problem types, but this study was only three weeks in duration. Learning to solve problems evolves gradually over a long period of time as is evidenced by the slow but continual growth in achievement on single step problems from grades three through six, so longer term studies are needed.

Finally the "Smaller Numbers" hint was also very helpful. However, asking children to transform a problem into a smaller numbers problem involves the difficult process of choosing "logical" numbers to replace the numbers in the given problem. Very probably, low ability students would have as much trouble doing this as solving the problem. The smaller

numbers technique is one, though, that might possibly be incorporated into textbooks. For example, pages of paired problems could be developed for which the student would be directed to try to solve the smaller numbers problem whenever he encountered difficulty on the larger numbers problem. Multiple step problems would be good candidates for this technique. Based on the results which have been reported in this paper we are quite confident that when using this paired problems approach, children could solve many problems that they would otherwise be unable to solve. We do not know though if children who were exposed to this teaching methodology would show improved achievement when the smaller numbers problem crutch was removed.

Asking children what they are supposed to find in a problem or what's given proved to be of little value. This confirms Burch's results (1953). Burch even claimed that making children answer these questions was detrimental to children's problem solving. Also of doubtful value is the methodology of presenting problems in low verbal form. Though low verbal problems helped very good students, we believe that rereading the problems would have been equally as effective.

The relationship of calculator usage to problem solving is unresolved by this study. Obviously a calculator does not help a child select the operation needed to solve a problem. Nevertheless there was some evidence at the third and fourth grade levels that low and average ability children performed better than expected on verbal problems because of calculator usage. This may simply be due to the fact that the instrument used to determine their problem solving ability, the ITBS test, does not adequately separate problem solving ability from computational proficiency. On the other hand it's possible that the calculator in some way frees the child

to focus his/her attention on problem relationships when the anxiety of computation is removed. We simply don't know, but the trend warrants further investigation in teaching studies.

With respect to rate problems, it seems fairly evident that instructional methods which focus the child's attention on the many-to-many correspondences in rate problems have a high probability of success. The unit rate method should probably be taught, but it has serious limitations and should not be taught exclusively.

The question of how to teach extraneous data problems was not addressed directly in this study. No data were collected on methods of helping children "process out" unnecessary information. It did seem, however, that many children who were troubled initially by extraneous information just ignored it once their attention had been focused by the hints on the significant attributes of the problem. In other words, if children could be taught better strategies of solving the usual single step and multiple step problems, their performance on extraneous data problems might also improve. There are many valuable suggestions in the literature for dealing directly with the extraneous data difficulty which should be given consideration when developing materials for teaching problem solving.

The Status of Current Practices of Teaching Verbal Problem Solving

As mentioned earlier, the investigator recently surveyed the verbal problem solving programs in five popular sixth grade mathematics textbooks. Teaching procedures were examined as well as the distribution of problem types for which the results were reported earlier. How did the instructional methods suggested by this study fare? In brief, not well!

The status in sixth grade textbooks of each of the problem solving strategies which, based on the results of this study, appear to have promise is examined below.

Drawings. Two of the textbooks provided exercises for which children had to read drawings but none taught children to make drawings.

Smaller numbers problems. One series provided mini-problems which contained very small numbers, but the smaller numbers problems were not used as a technique for helping children solve problems with larger numbers.

Focusing on the action. This technique was never explicitly taught. It may have been intended in some of the exercises where children were asked to write missing addend and missing factor equations (as for example $35 + n = 49$ for an additive subtraction problem which has a joining action). Three of the textbooks taught "writing equations" but no more than three pages per book were devoted to the topic. Teachers were never told to direct the problem solvers attention to the action.

Focusing on part-part-whole relationships. This strategy, also, was never used explicity. Equation writing instruction may have had as its intent a focus on part-part-whole, but the comments made above about the teaching of equation writing apply as well to part-part-whole instruction.

Focusing on the size of the answer. Four of the textbooks encouraged children to estimate the answers to problems, but this is different from the "size of answer" hint used in the study. In order to estimate the child must already know what operation is needed to solve the problem. The question, "Is the answer larger than _____? (or smaller than _____?)," helped children determine the operation.

Although there was very little evidence that textbooks utilized any of the strategies that were found to be helpful, those methods which seemed to be of doubtful value were in evidence.

Low verbal content. Three of the five textbooks provided low verbal content problems.

Question asks? and What's Given. Two out of the five textbook series directed children to ask themselves "What do I know?" and "What must I find?" or a similar pair of questions as the first steps in solving a problem. Since the next step in both textbooks was deciding on the computation there is an implication that answering the two questions will lead children to the necessary operations. According to our results, they do not.

It is unsettling, to say the least, to find that the results of research and classroom practice as evidenced in textbooks are so divergent. Yet without further long-term studies where the problem solving strategies pointed to by this study are examined in the classroom, one must be very cautious about either condemning standard practice or promoting the practices which have been suggested here.

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APPENDIX A

PROBLEM SETS FOR GRADES THREE, FOUR FIVE AND SIX

THIRD GRADE PROBLEMS

Single Step Problems

Addition

Frank spent 19¢ for a candy bar. He also spent 25¢ for a package of balloons. How much did Frank spend?

Ans. 44¢

"Take Away" Subtraction

Mary had 45 stamps in her collection. She gave 18 to her friend Jane. How many does Mary have now?

Ans. 27 stamps

Comparative Subtraction

Continuous: It took Bob 17 minutes to walk to the swimming pool. It takes Kim 35 minutes to walk to the swimming pool. How much longer does it take Kim?

Ans. 18 minutes

Discrete: There are 47 passengers on a bus. If there are 65 seats on the bus, how many seats are empty?

Ans. 18 seats

Additive Subtraction

Continuous: Sally is riding her bicycle in a race. She has gone 16 miles. The race is 25 miles long. How much further does she still have to go?

Ans. 9 miles

Discrete: Sue has 35 cents. She is saving to buy a Star Wars poster that costs 49 cents. How much money does she still have to save?

Ans. 14 cents

Multiplication

Tom wants to buy 3 apples. They cost 6¢ each. How much will he spend?

Ans. 18¢

Measurement Division - No Remainder

Continuous: Mary uses 3 yards of ribbon to make a bow. How many bows can she make from 24 yards of ribbon?

Ans. 8 bows

Discrete: Miss Jensen is putting 24 pencils into boxes. She puts 3 pencils in a box. How many boxes does she need?

Ans. 8 boxes

Measurement Division - With Remainder

Continuous: The librarian bought 29 feet of Contact paper to cover some book shelves. It takes 8 feet of Contact paper to cover a shelf. How many shelves can be covered with the Contact paper?

Ans. 3 shelves (3 shelves, 5 feet of Contact paper left over)

Discrete: The librarian wants to move 29 books into the reading clinic room. Each student can carry 8 books. How many students will be needed to carry the books?

Ans. 4 students (3 students, 5 books left over)

Partition Division - No Remainder

Continuous: Jim wrapped 8 packages and sealed them with Scotch tape. Altogether he used 24 inches of tape. He used the same amount of tape on each package. How much Scotch tape did he use on a package?

Ans. 3 inches

Discrete: At the factory where tennis balls are made it took 8 boxes to pack 24 tennis balls. How many tennis balls were in each box?

Ans. 3 tennis balls

Partition Division - With Remainder

Continuous: John has 10 yards of ribbon to wrap 4 packages. He will use the same amount of ribbon for each package. How much ribbon does he use on each package?

Ans. 2 1/2 yards (2, remainder 2)

Discrete: Mr. Jones has 10 Hershey bars to give to 4 children. He wants to share the candy equally among the children. How many Hershey bars should he give to each child?

Ans. 2 1/2 candy bars (2 remainder 2)

Extraneous Data

Same units: There are 24 people in line at a movie theater. In the theater there are 18 empty seats. There are 6 seats in each row. How many people will not get a seat?

Ans. 6 people

Unlike units: A 6 ounce can of frozen lemonade costs 20¢. If it takes 4 cans to make enough lemonade for my picnic jug, how much does it cost to fill my jug with lemonade?

Ans. 80¢

Multiple Step Problems

Using Multiplication

At a bake sale Martin bought 4 cookies at 6¢ each and 4 doughnuts at 9¢ each. How much did he spend?

Ans. 60¢

Using Division

There are 27 girls and 36 boys in the Third Grade. Mr. Allen put 9 children on a team for a relay race. How many teams did he have?

Ans. 7 teams

Extraneous Data

Betty has a 10 page photo album. On each page there is room for 4 pictures. She has 24 pictures of which 8 pictures are from her trip to Chicago. How many pictures does she still need to fill her photo album?

Ans. 16 pictures

Rate Problems

Unit Rate a Whole Number

Continuous: If Mary can ride her bicycle 2 miles in 20 minutes, how long will it take her to ride 10 miles?

Ans. 100 minutes

Discrete: Mary can buy 2 sticks of candy for 10¢. How much will 3 sticks of candy cost?

Ans. 15¢

Unit Rate a Fractional Number

Continuous: If Joe can ride his bike 2 miles in 15 minutes, how long will it take him to ride 8 miles?

Ans. 60 minutes

Discrete: The cost of 2 pencils is 15¢. How much will 6 pencils cost?

Ans. 45¢

FOURTH GRADE PROBLEMS

Single Step Problems

Addition

The Turner family's new washing machine cost $423. Their dryer cost $354. How much did they spend for laundry equipment?

Ans. $777

"Take Away" Subtraction

Mary had 153 stamps in her collection. She sold 42 of them to her friend Jane. How many does Mary have now?

Ans. 111 stamps

Comparative Subtraction

Continuous: Bob's mother drove to Des Moines in 137 minutes. It took Betty's mother 152 minutes to drive to Des Moines. How much longer does it take Betty's mother?

Ans. 15 minutes

Discrete: There are 137 passengers on a plane. If there are 152 seats on the plane, how many seats are empty?

Ans. 15 seats

Additive Subtraction

Continuous: Sally's Scout troop is on a bicycle trip. They have travelled 75 miles. The trip is 98 miles in all. How much further do they have to go?

Ans. 23 miles

Discrete: Sue has 75 cents. She is saving to buy a Star Wars poster that costs 98 cents. How much money does she still have to save?

Ans. 23¢

Multiplication

Tom is buying apples for each of the 12 boys in his Cub Scout troop. Apples cost 25¢ each. How much must Tom pay for the apples?

Ans. $3.00

Measurement Division - No Remainder

Continuous: Miss Jensen is making skip ropes that are 6 feet long. She is cutting the skip ropes from a 732 foot roll of rope. How many skip ropes can she make?

Ans. 122 skip ropes

Discrete: Miss Jensen is putting 732 pencils into boxes. She puts 6 pencils in a box. How many boxes does she need?

Ans. 122 boxes

Measurement Division - With Remainder

Continuous: The librarian bought 150 feet of Contact paper to cover some book shelves. It takes 8 feet of Contact paper to cover a shelf. How many shelves can be covered with the Contact paper?

Ans. 18 shelves (18 shelves, 6 feet left over)

Discrete: The librarian wants to move 150 books into the reading clinic room. Each student can carry 8 books. How many students will be needed to carry the books?

Ans. 19 students (18 students, 6 books left over)

Partition Division - No Remainder

Continuous: Jim is making 6 kite strings from a 432 foot ball of string. Each kite string will be the same length. How long should he make the kite string?

Ans. 72 feet

Discrete: At the factory where tennis balls are made it took 6 large boxes to pack 1650 balls. How many tennis balls were in each box?

Ans. 275 balls

Partition Division - With Remainder

Continuous: Mr. Jones has 75 yards of ribbon to wrap 30 packages for the children in his class. He will use the same amount of ribbon for each package. How much ribbon does he use on a package?

Ans. 2 1/2 yards (2, remainder 15)

Discrete: Mr. Jones has 75 Hershey bars for the 30 children in his class. He wants to share the candy equally among the children. How many Hershey bars should he give to each child?

Ans. 2 1/2 Hershey bars (2, remainder 15)

Extraneous Data

Same Units: There are 480 people in line at a movie theater. The theater seats 456 people, with 24 seats in each row. How many people will not get a seat?

Ans. 24 people

Unlike units: A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade to fill my picnic jug, how much does it cost to fill my jug with lemonade?

Ans. $1.92

Multiple Step Problems

Using Multiplication

At a bake sale, Mrs. Lucas bought 12 cookies at 15¢ each and 12 doughnuts at 18¢ each. How much did she spend?

Ans. $3.96

Using Division

There are 60 girls and 72 boys in the Fourth Grade. Mr. Allen put 12 children on a team for a relay race. How many teams did he have?

Ans. 11 teams

Extraneous Data

Betty has an 80 page photo album. On each page there is room for 20 pictures. She has 800 pictures of which 160 are from her trip to Hawaii. How many pictures does she still need to fill her photo album?

Ans. 800 pictures

Rate Problems

Unit Rate a Whole Number

Continuous: If Mary can ride her bicycle 2 miles in 20 minutes, how long will it take her to ride 10 miles?

Ans. 100 minutes

Discrete: Mary can buy 3 sticks of candy for 15¢. How much will 7 sticks of candy cost?

Ans. 35¢

Unit Rate a Fractional Number

Continuous: If Joe can ride his bike 2 miles in 15 minutes, how long will it take him to ride 8 miles?

Ans. 60 minutes

Discrete: The cost of 2 pencils is 15¢. How much will 6 pencils cost?

Ans. 45¢

FIFTH GRADE PROBLEMS

Single Step Problems

Addition

Bob's new shirt cost $8.99. His new shoes cost $13.49. How much did Bob spend?

Ans. $22.48

"Take Away" Subtraction

Mary had 250 stamps in her collection. She sold 89 of them to her friend Jane. How many does Mary have now?

Ans. 161 stamps

Comparative Subtraction

Continuous: Bob's mother drove to Council Bluffs in 258 minutes. It took Betty's mother 300 minutes to drive to Council Bluffs. How much longer did it take Betty's mother?

Ans. 42 minutes

Discrete: There are 257 passengers on a jumbo jet. If there are 310 seats on the plane, how many seats are empty?

Ans. 53 seats

Additive Subtraction

Continuous: Sally's family is on a trip. So far they have driven 823 miles. The total distance they have to drive is 1295 miles. How much further do they still have to go?

Ans. 472 miles

Discrete: Sue has $82.25. She is saving to buy a new bicycle that costs $129.59. How much money does she still have to save?

Ans. $47.34

Multiplication

Mr. Smith is buying apples for each of the 28 children in his class. Apples cost 25¢ each. How much must Mr. Smith pay for the apples?

Ans. $7.00

Measurement Division - No Remainder

Continuous: Miss Jensen has an 864 foot ball of yarn. The children in her class are making Christmas tree decorations from the yarn. They use 24 feet of yarn for each decoration. How many decorations can they make?

Ans. 36 decorations

Discrete: Miss Jensen is putting 864 pencils into boxes. She puts 24 pencils in a box. How many boxes does she need?

Ans. 36 boxes

Measurement Division - Remainder

Continuous: The librarian bought 350 feet of Contact paper to cover the book cases in the library. It takes 15 feet of paper to cover a case. How many book cases can be covered with the Contact paper?

Ans. 23 book cases (23 cases, 5 feet left over)

Discrete: The librarian wants to move 350 books into the reading clinic room. Each student can carry 15 books. How many students will be needed to carry books?

Ans. 24 students (23 students, 5 books left over)

Partition Division - No Remainder

Continuous: Jim's Scout troop is making 18 kite strings from a 432 yard ball of string. Each kite string will be the same length. How long should they make the kite strings?

Ans. 24 yards

Discrete: At the factory where tennis balls are made it took 18 boxes to pack 432 tennis balls. How many tennis balls were in each box?

Ans. 24 balls.

Partition Division - With Remainder

Continuous: Mr. Jones has 72 yards of ribbon to wrap 27 packages for the children in his class. He will use the same amount of ribbon for each package. How much ribbon does he use on a package?

Ans. 2 2/3 yards (2, remainder 18)

Discrete: Mr. Jones has 72 Hershey bars for the 27 children in his class. He wants to share the candy equally among the children. How many Hershey bars should he give to each child?

Ans. 2 2/3 Hershey bars (2, remainder 18)

Extraneous Data

Same units: There are 1125 people in line at a movie theater. The theater seats 975 people with 25 seats in each row. How many people will not get a seat?

Ans. 150 people

Unlike Units: A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade to fill my picnic jug, how much does it cost to fill my jug with lemonade?

Ans. $1.92

Multiple Step Problems

Using Multiplication

A plane is making a trip of 4,300 miles. During the first 7 hours of the trip it travelled at the rate of 570 miles per hour. How much further does it still have to go?

Ans. 310 miles

Using Division

The 62 pupils in the Sixth Grade share the cost of a museum visit. A bus can be rented from the Ajax Bus Company for $37.20. A bus can be rented from the Blue Goose Bus Company for $34.10. How much would each pupil save by renting the bus from the Blue Goose Company?

Ans. 5¢ (or $.05)

Extraneous Data

Betty has an 80 page photo album. On each page there is room for 20 pictures. She has 800 pictures of which 160 are from her trip to Hawaii. How many pictures does she still need to fill her photo album?

Ans. 800 pictures

Rate Problems

Unit Rate a Whole Number

Continuous: If Mary can ride her bicycle 2 miles in 20 minutes, how long will it take her to ride 9 miles?

Ans. 90 minutes

Discrete: Mary can buy 3 sticks of candy for 15¢. How much will 7 sticks of candy cost?

Ans. 35¢

Unit Rate a Fractional Number

Continuous: If Joe can ride his bike 2 miles in 15 minutes, how long will it take him to ride 8 miles?

Ans. 60 minutes

Discrete: The cost of ping pong balls is 85¢. How much will 12 ping pong balls cost?

Ans. $3.40

SIXTH GRADE PROBLEMS

Single Step Problems

Addition

Bob's new shirt cost $8.99. His new shoes cost $13.49. How much did Bob spend?

Ans. $22.48

"Take Away" Subtraction

Mary had 1275 stamps in her collection. She sold 698 to her friend Jane. How many does Mary have now?

Ans. 577 stamps

Comparative Subtraction

Continuous: Bob's mother drove to Council Bluffs in 258 minutes. It took Betty's mother 300 minutes to drive to Council Bluffs. How much longer did it take Betty's mother?

Ans. 42 minutes

Discrete: There are 257 passengers on a jumbo jet. If there are 310 seats on the plane, how many seats are empty?

Ans. 53 seats

Additive Subtraction

Continuous: Sally's family is on a trip. So far they have driven 1984 miles. The total distance they have to drive is 3100 miles. How much further do they still have to go?

Ans. 1116 miles

Discrete: Sue's sister, Judy, has $1984. She is saving to buy a new car for which she needs $3100. How much more does she still have to save?

Ans. $1116

Multiplication

A department store sold 120 television sets at $312 each. How much money did the store receive?

Ans. $37,440

Measurement Division - No Remainder

Continuous: Betty's father sells dog pens that he makes from fence wire. He uses 144 feet of fence to make a pen. How many pens can he make from 864 feet of fence wire?

Ans. 6 pens

Discrete: Miss Jensen is putting 864 pencils into boxes. She puts 144 pencils in a box. How many boxes does she need?

Ans. 6 boxes

Measurement Division - With Remainder

Continuous: The librarian bought 350 feet of Contact paper to cover the book cases in the library. It takes 15 feet of paper to cover a case. How many book cases can be covered with the Contact paper?

Ans. 23 book cases (23 cases, 5 feet left over)

Discrete: The librarian wants to move 350 books into the reading clinic room. Each student can carry 15 books. How many students will be needed to carry books?

Ans. 24 students (23 students, 5 books left over)

Partition Division - No Remainder

Continuous: Jim's Scout troop is making 18 kite strings from a 432 yard ball of string. Each kite string will be the same length. How long should they make the kite strings?

Ans. 24 yards

Discrete: At the factory where tennis balls are made it took 18 boxes to pack 432 tennis balls. How many tennis balls were in each box?

Ans. 24 balls

Partition Division - With Remainder

Continuous: Mr. Jones has 270 feet of ribbon to wrap 24 packages for the children in his class. He will use the same amount of ribbon for each package. How much ribbon does he use on a package?

Ans. 11 1/4 feet (11, remainder 6)

Discrete: Mr. Jones works at a bakery. He has 270 apples from which he must make 24 apple pies. He will use the same number of apples in each pie. How many apples should he put in each pie?

Ans. 11 1/4 apples (11, remainder 6)

Extraneous Data

Same units: There are 1125 people in line at a movie theater. The theater seats 975 people with 25 seats in each row. How many people will not get a seat?

Ans. 150 people

Unlike units: A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade to fill my picnic jug, how much does it cost to fill my jug with lemonade?

Ans. $1.92

Multiple Step Problems

Using Multiplication

A plane is making a trip of 4,300 miles. During the first 7 hours of the trip it travelled at the rate of 570 miles per hour. How much further does it still have to go?

Ans. 310 miles

Using Division

The 62 pupils in the Sixth Grade share the cost of a museum visit. A bus can be rented from the Ajax Bus Company for $37.20. A bus can be rented from the Blue Goose Bus Company for $34.10. How much would each pupil save by renting the bus from the Blue Goose Company?

Ans. 5¢ (or $.05)

Extraneous Data

Betty has a 75 page photo album. On each page there is room for 25 pictures. Betty has 1250 pictures of which 125 are from her trip to Hawaii. How many pictures does she still need to fill her photo album?

Ans. 625 pictures

Rate Problems

Unit Rate a Whole Number

Continuous: If Mary can ride her bicycle 2 miles in 20 minutes, how long will it take her to ride 9 miles?

Ans. 90 minutes

Discrete: Mary can buy 3 sticks of candy for 15¢. How much will 7 sticks of candy cost?

Ans. 35¢

Unit Rate a Fractional Number

Continuous: If Joe can ride his bike 2 miles in 15 minutes, how long will it take him to ride 8 miles?

Ans. 60 minutes

Discrete: The cost of 3 ping pong balls is 85¢. How much will 12 ping pong balls cost?

Ans. $3.40

APPENDIX B

SAMPLE INTERVIEW CODING FORMS AND CHILDREN'S PROBLEM PAGES

Interviewer _____ Date _____

Student _____ School _____ City _____

Problem Type: PARTITION DIVISION - NO REMAINDER Grade 3 J-D
Discrete units

At the factory where tennis balls are made it took 8 boxes to pack 24 tennis balls. How many tennis balls were in each box?

Ans. 3 tennis balls

ANSWER ___ Right ___Wrong

___Immediate use of correct operation.
___Wrong (computation, process) but correct after reconsidering; invoked by (student, teacher).
___Wrong, but correct after interviewer hints.
___Couldn't get started, but correct after hints.
___Other:_____

___Process correct, wrong computation.
___Process wrong.
___No attempt made (couldn't get started).
___Randomly computes with numbers in problem.
___Other: _____

MODE OF SOLUTION: ___paper and pencil ___mental ___calculator
___manipulative ___drawings

INTERPRETATION: ___correct label ___wrong label (give label:_____)

OPERATION: ___Division ___Repeated addition
___Multiplication ___Repeated subtraction
___Used many-to-one correspondence ___Counting

FOCUS: ___Determined from "thinking out loud". ___Child's description of what he did after work was completed.

Describe how child made choice of operations:

OR: Check hints given and circle the one(s) that helped.

Mode hints: ___Would the calculator help?
___How about using the sticks?
___How about making a drawing?

Focus hints: ___(a) Let's see if you could solve the problem when it's stated in another way. Turn down the top of your paper and read what you see there.
___(b) Let's see if you could solve a problem just like this one with smaller numbers. Fold up the bottom of your paper and read what you see there.
___(c1) Were bunches of tennis balls taken away from the 24 balls or were bunches of tennis balls put together?_____ How many bunches? ____ When you take away 8 bunches of tennis balls and each bunch is the same size, what should you do to solve the problem?_____
___(c2) Give drawing hint, then: Try to show both the tennis balls and the boxes in your drawing.
___(d) Are you given the total number of tennis balls or are you looking for the total?_____ Do you know the number of tennis balls in each part?_____ Do you know the number of parts?_____ When you know the total number of tennis balls and you know how many parts there are, what should you do to solve the problem?_____
___(e) Will the answer be larger or smaller than 24?_____ What could you do to get a smaller number?_____
___(f) What's the question asking for?_____
___(g) What's given in the problem?_____
___(h) Personalize.

J-D

At the factory where tennis balls are made, it took 8 boxes to pack 24 tennis balls. How many tennis balls were in each box?

At the factory where tennis balls are made it took 3 boxes to pack 12 tennis balls. How many tennis balls were in each box?

Have 24 tennis balls.
Put them into 8 boxes.
How many in a box?

Interviewer _____ Date _____

Student _____ School _____ City _____

Problem Type: MULTIPLE STEP
Multiplication and Addition

Grade 4 L

At a bake sale, Mrs. Lucas bought 12 cookies at 15¢ each and 12 doughnuts at 18¢ each. How much did she spend?

Ans. $3.96

ANSWER ___ Right ___Wrong

___Immediate use of correct operation.
___Wrong (computation, process) but correct after reconsidering; invoked by (student, teacher).
___Wrong, but correct after interviewer hints.
___Couldn't get started, but correct after hints.
___Other: _____

___Process correct, wrong computation.
___Process wrong.
___No attempt made (couldn't get started).
___Randomly computes with numbers in problem.
___Other: _____

MODE OF SOLUTION: ___paper and pencil ___mental ___calculator
___manipulative ___drawings

INTERPRETATION: ___correct label ___wrong label (give label: _____)

MULTIPLE STEP OPERATIONS:
___Logical order of operations. Order: _____
___One step correct but one or more steps wrong. Correct step: _____
___Number Crunching
___Other: _____

FOCUS: ___Determined from "thinking out loud". ___Child's description of what he did after work was completed.

Describe how child made choice of operation:

OR: Check hints given and circle the one(s) that helped.

Mode Hints: ___Would the calculator help?
___How about using the sticks?
___How about making a drawing?

Focus Hints:
___(a) Let's see if you could solve the problem when it's stated in another way. Turn down the top of your paper and read what you see there.
___(b) Let's see if you could solve a problem just like this one with smaller numbers. Fold up the bottom of your paper and read what you see there.
___(c) Are you putting together amounts of money? _____ What amounts of money are you putting together? _____ When you put the cost of the cookies and the cost of the doughnuts together, what should you do to solve the problem? _____
___(d) Are you given the total amount of money Mrs. Lucas spent or are you looking for the total? _____ Do you know the partial amounts of money Mrs. Lucas spent? _____ How could you find the partial amounts? _____ When you know the partial amounts Mrs. Lucas spent and you want to find the total what should you to to solve the problem? _____
___(e) Will the cost of the cookies be larger or smaller than 15¢? _____ Will the cost of the doughnuts be larger or smaller than 18¢ _____ What could you do to get a larger number? _____
___(f) What's the question asking for? _____
___(g) What's given in the problem? _____
___(h) Personalize.

L

At a bake sale, Mrs. Lucas bought 12 cookies at 15¢ each and 12 doughnuts at 18¢ each. How much did she spend?

At the bake sale Mrs. Lucas bought 3 cookies at 5¢ each and 3 doughnuts at 7¢ each. How much did she spend?

Bought 12 cookies at 15¢ each.
Bought 12 doughnuts at 18¢ each.
How much money was spent?

Interviewer _____ Date _____

Student _____ School _____ City _____

Problem Type: IRRELEVANT DATA - UNLIKE UNITS
Multiplication

Grade 4 N-U

A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade to for my picnic jug, how much does it cost to fill my jug with lemonade?

ANSWER ___ Right ___ Wrong

___Immediate use of correct operation.
___Wrong (computation, process) but correct after reconsidering; invoked by (student, teacher).
___Wrong, but correct after interviewer hints.
___Couldn't get started, but correct after hints.
___Other:_____

___Process correct, wrong computation.
___Process wrong.
___No attempt made (couldn't get started).
___Randomly computes with numbers in problem.
___Other: _____

MODE OF SOLUTION: ___paper and pencil ___mental ___calculator
___manipulative ___drawings

INTERPRETATION: ___correct label ___wrong label (give label:_____)

IRRELEVANT DATA: ___Ignored irrelevant data.
___Attempted to use irrelevant data, but rejected.
___Insisted on computation involving irrelevant data.

OPERATION: ___Multiplication ___Division ___Counting ___Repeated Addition
___Used many-to-one correspondence ___Other:_____

FOCUS: ___Determined from "thinking out loud". ___Child's description of what he did after work was completed.

Describe how child made choice of operation:

OR: Check hints given and circle the one(s) that helped.

Mode hints: ___Would the calculator help?
___How about using the sticks?
___How about making a drawing?

Focus hints: ___(a) Let's see if you could solve the problem when it's stated in another way. Turn down the top of your paper and read what you see there.
___(b) Let's see if you could solve a problem just like this one with smaller numb rs. Fold up the bottom of your paper and read what you see there.
___(c) What must you put together to solve the problem?_____
___(d) Are you given the total cost or must you find the total?_____ What parts of this total are given?_____ When you know several parts that are the same amount, what do you do to find the total?
_____
___(e) Will the answer be larger or smaller than 24¢?_____ What could you do to get a larger amount?_____
___(f) What's the question asking for?_____
___(g) What's given in the problem?_____
___(h) Personalize.

N-U

A 6 ounce can of frozen lemonade costs 24¢. If it takes 8 cans to make enough lemonade for my picnic jug, how much does it cost to fill my jug with lemonade?

A 6 ounce can of frozen lemonade costs 10¢. If it takes 3 cans to make enough lemonade for my picnic jug, how much does it cost to fill my jug with lemonade?

6 ounce can of lemonade costs 24¢.

Need 8 cans.

How much will lemonade cost?

Interviewer _____ Date _____

Student _____ School _____ City _____

Problem Type: RATE - UNIT PRICE CANNOT BE COMPUTED Grade 5 F-c
Continuous Quantities

If Joe can ride his bike 2 miles in 15 minutes, how long will it take him to ride 8 miles?

Answer: 60 minutes

ANSWER ___ Right ___Wrong

___Immediate use of correct operation.
___Wrong (computation, process) but correct after reconsidering; invoked by (student, teacher).
___Wrong, but correct after interviewer hints.
___Couldn't get started, but correct after hints.
___Other: _____

___Process correct, wrong computation.
___Process wrong.
___No attempt made (couldn't get started).
___Randomly computes with numbers in problem.
___Other: _____

MODE OF SOLUTION: ___paper and pencil ___mental ___calculator
___manipulative ___drawings

INTERPRETATION: ___correct label ___wrong label (give label: _____)

OPERATION: ___Multiplication ___Division ___Counting ___Repeated addition
Order of operations: _____

RATE STRUCTURE: ___Appeared to use many-to-many correspondence.
___Did not use correspondence. Focused on arithmetic operations.
___Other: _____

FOCUS: ___Determined from "thinking out loud". ___Child's description of what he did after work was completed.

Describe how child made choice of operation:

OR: Check hints given and circle the one(s) that helped.

Mode Hints: ___Would the calculator help?
___How about using the sticks?
___How about making a drawing?

Focus Hints: ___(a) Let's see if you could solve the problem when it's stated in another way. Turn down the top of your paper and read what you see there..
___(b) Let's see if you could solve a problem just like this one with smaller numbers. Fold up the bottom of your paper and read what you see there.
___(c) Give drawing hint, then: Try to show both the miles and the minutes on your drawing.
___(d) Can you tell me how long it takes Joe to ride 1 mile? _____
If it takes 7 1/2 minutes to ride 1 mile, how could you find out how long it will take to ride 8 miles? _____
___(e) Will the answer be larger or smaller than 15 minutes? _____
What could you do to get a larger number? _____
___(f) What's the question asking for? _____
___(g) What's given in the problem? _____
___(h) Personalize.

F-c

If Joe can ride his bike 2 miles in 15 minutes, how long will it take him to ride 8 miles?

If Joe can ride his bike 2 miles in 5 minutes,
how long will it take him to ride 6 miles?

2 miles in 15 minutes.
8 miles in how many minutes?

Interviewer _____ Date _____

Student _____ School _____ City _____

Problem Type: IRRELEVANT DATA - MULTIPLE STEP Grade 6 0
Multiplication, Additive Subtraction

Betty has a 75 page photo album. On each page there is room for 25 pictures. Betty has 1250 pictures of which 125 are from her trip to Hawaii. How many pictures does she still need to fill her photo album?

Ans. 625 pictures

ANSWER ___ Right ___Wrong

___Immediate use of correct operation.
___Wrong (computation, process) but correct after reconsidering; invoked by (student, teacher).
___Wrong, but correct after interviewer hints.
___Couldn't get started, but correct after hints.
___Other:_____

___Process correct, wrong computation.
___Process wrong.
___No attempt made (couldn't get started).
___Randomly computes with numbers in problem.
___Other: _____

MODE OF SOLUTION: ___paper and pencil ___mental ___calculator
___manipulative ___drawings

INTERPRETATION: ___correct label ___wrong label (give label:_____)

IRRELEVANT DATA: ___Ignored irrelevant data.
___Attempted to use irrelevant data, but rejected.
___Insisted on computation using irrelevant data.

MULTIPLE STEP OPERATIONS:
___Logical order of operations. Order:_____
___One step correct but one or more steps wrong. Correct step:_____
___Number crunching.
___Other:_____

FOCUS: ___Determined from "thinking out loud". ___Child's description of what he did after work was completed.

Describe how child made choice of operation:

Or: Check hints given and circle the one(s) that helped.
Mode hints: ___Calculator ___Stick ___Drawing

Focus hints:
___(a) Let's see if you could solve the problem when it's stated in another way. Turn down the top of your paper and read what you see there.
___(b) Let's see if you could solve a problem just like this one with smaller numbers. Fold up the bottom of your paper and read what you see there.
___(c) If Betty had filled her album, how many times would she have pasted 25 pictures on a page?_____ When you paste 25 pictures 75 times, what should you do to find the number of pictures in all?_____
___(1) Do you have to put some more pictures with Betty's 1250 pictures or take some pictures away from the 1250?_____ When you have to put some pictures with the 1250, what should you do to solve the problem?_____
___(2) Do you have to take some pictures away from the 1875 pictures needed to fill the album or put some pictures with the 1875?_____ When you take some pictures away from the 1875, what must you do to solve the problem?_____
___(d) Are you given the total number of pictrues in the album or must you find the total?_____ What is the total?_____ Are you given any part of this total?_____When you know the total and you know one of the parts, what must you do to solve the problem?_____
___(e) Will the answer be larger or smaller than the total number of pictures in the album?_____ What's the total?_____How could you get a number smaller than 1875?_____
___(f) What's the question asking for?_____
___(g) What' given in the problem?_____ ___(h) Personalize

Betty has a 75 page photo album. On each page there is room for 25 pictures. Betty has 1250 pictures of which 125 are from her trip to Hawaii. How many pictures does she still need to fill her photo album?

Betty has a 10 page photo album. On each page there is room for 3 pictures. She has 20 pictures of which 8 are from her trip to Hawaii. How many pictures does she still need to fill her album?

Photo album has 75 pages.
Each page holds 25 pictures.
Have 1250 pictures of which 125 are pictures of Hawaii.
How many pictures still needed to fill album?